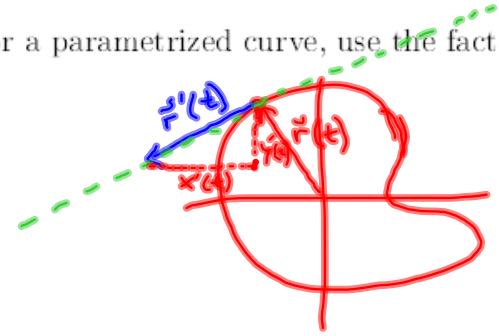
1 3.9: Tangents of Parametrized Curves

To find the slope of the tangent line for a parametrized curve, use the fact that

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$



Horizontal Tangent : $\frac{dy}{dx} = 0$

$\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$

Vertical Tangent : $\frac{dy}{dx} \to \infty$

$\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$

$\vec{r}'(t) = x'(t)\vec{i} + y'(t)\vec{j}$

$m = \frac{y'(t)}{x'(t)}$

Oct 14-12:31 PM

Examples:

Find an equation of the line tangent to the curve given by $x = 3\cos t$, $y = 4\sin t$ at the point where $t = \frac{\pi}{6}$.

Slope $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4\cos t}{-3\sin t}$

$m = \frac{4\cos \pi/6}{-3\sin \pi/6} = \frac{4\left(\frac{\sqrt{3}}{2}\right)}{-3\left(\frac{1}{2}\right)} = -\frac{4\sqrt{3}}{3}$

Point $t = \frac{\pi}{6}$, so $x = 3\cos\frac{\pi}{6} = \frac{3\sqrt{3}}{2}$ $\left(\frac{3\sqrt{3}}{2}, 2\right)$

$y = 4\sin\pi/6 = 2$

Equation $\boxed{y - 2 = \frac{-4\sqrt{3}}{3}\left(x - \frac{3\sqrt{3}}{2}\right)}$

Oct 14-12:32 PM

point $(0,4)$ (x, y)

The curve $x = t^3 - 4t$, $y = t^2$ crosses itself at the ~~origin~~. Find equations of both tangent lines.

Slope $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{3t^2-4}$

Find t:

$t^3 - 4t = 0$ $\quad t^2 = 4$

$t(t^2-4) = 0$ $\quad t = \pm 2$

~~$t=0$~~, $t = \pm 2$

Must satisfy both equations

$(t=2)$

$m_1 = \frac{2(2)}{3(2)^2-4} = \frac{4}{8} = \frac{1}{2}$

equation 1 $\boxed{y = \frac{1}{2}x + 4}$

$(t=-2)$

$m_2 = \frac{2(-2)}{3(-2)^2-4} = \frac{-4}{8} = \frac{-1}{2}$

equation 2 $\boxed{y = -\frac{1}{2}x + 4}$

(On your own): Find the points on the curve $x = t^3 - 3t^2$, $y = t^3 - 3t$ where the tangent line is
a) horizontal
b) vertical

b) $\frac{dx}{dt} = 0$

$3t^2 - 6t = 0$

$3t(t-2) = 0$

$t = 0 \quad t = 2$

Points

$t=0$ $x = 0$ $y = 0$ $(0,0)$

$t=2$ $x = 2^3 - 3\cdot 2^2 = -4$ $y = 2^3 - 3\cdot 2 = 2$ $(-4,2)$

a) $\frac{dy}{dt} = 0$

$3t^2 - 3 = 0$

$t = \pm 1$

Points

$t=1$ $x = 1^3 - 3\cdot 1^2 = -2$ $y = 1^3 - 3\cdot 1 = -2$ $(-2,-2)$

$t=-1$ $x = (-1)^3 - 3(-1)^2 = -4$ $y = (-1)^3 - 3(-1) = 2$ $(-4,2)$

Oct 14-12:32 PM