1 4.2: Inverse Functions

functions vs. one-to-one functions:

If $f$ is one-to-one, the inverse of $f$ is a function $f^{-1}$ such that

If $(a, b)$ is on the graph of $y = f(x)$, then

If $f$ is one-to-one and differentiable at $x = g(a)$, where $g = f^{-1}$, then
Examples:

Show \( f(x) = \frac{2 - x}{2 + x} \) is one-to-one and find \( f^{-1} \).

TRUE OR FALSE? The inverse of \( f(x) = \sqrt{x - 1} \) is \( g(x) = x^2 + 1 \). Explain.
Given $g$ is the inverse of $f(x) = x^5 - x^3 + 4x$, find $g'(4)$.

(On your own): The function $f(x) = \tan x$ is one-to-one on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. If $g = f^{-1}$, find $g'(1)$.