1 4.3: Review of Logarithmic Functions

Definition and Properties of Logarithms

$y = \log_a x$ means $x = a^{y}$ ← what exponent?

$\log_3 \frac{1}{27} = y$ means

$3^{y} = \frac{1}{27} = 3^{-3}$

$y = -3$

$y = \ln x$ means $x = e^{y}$

$\log_e$

1 log "hard"

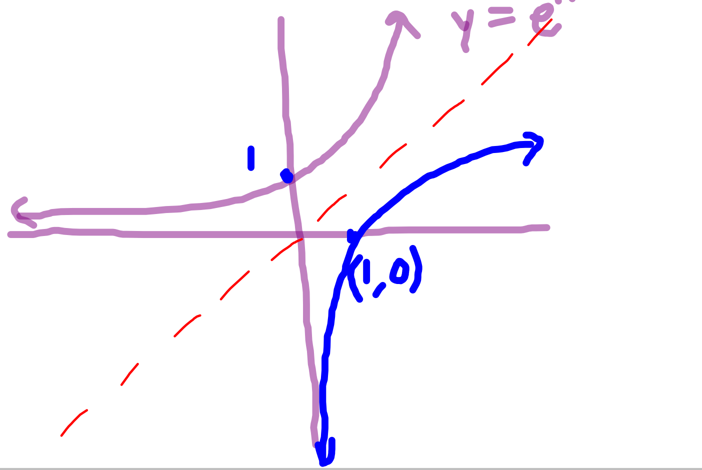
mult logs "easy"

On Valid Domains

1. $\log_a(xy) = \log_a x + \log_a y$
2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_a(x^n) = n \log_a x$

$\cancel{\log_a(x+y) =}$

Graphs of Logarithmic Functions:



V.A. $x=0$ $\lim_{x\to 0^+} \ln x = -\infty$

$\lim_{x\to \infty} \ln x = \infty$

Domain: $x > 0$

Examples:

Compute $\log_5 10 + \log_5 20 - 3\log_5 2$.

$= \log_5 10 + \log_5 20 - \log_5 2^3$

$= \log_5 (10 \cdot 20) - \log_5 8$

$= \log_5 \left(\frac{200}{8}\right)$

$= \log_5 (25)$

$= \boxed{2}$ since $5^{2} = 25$

The formula to compute the amount of money A in an account earning $100r\%$ interest compounded m times per year after t years is $A = P\left(1+\frac{r}{m}\right)^{mt}$. If \$10,000 are invested in a CD earning 4% per year compounded monthly, when will the account have \$15,000?

$P = 10000$
$r = .04$
$m = 12$
$A = 15000$

$\frac{15000}{10000} = \frac{\cancel{10000}}{\cancel{10000}}\left(1+\frac{.04}{12}\right)^{12t}$ exponent

$\ln 1.5 = \ln\left(1+\frac{.04}{12}\right)^{12t}$ 1.003333

Idea!: $\log_{1.0033} 1.5 = 12t$

$\ln 1.5 = 12t \cdot \ln(1.00333)$

$\frac{\ln 1.5}{\ln 1.0033} = 12t$

$t = \frac{1}{12}\frac{\ln 1.5}{\ln 1.0033} \approx \boxed{10.15 \text{ years}}$

NOTE $\log_{1.0033} 1.5 = \frac{\ln 1.5}{\ln 1.0033}$

Change of Base Formula

$$\boxed{\log_a x = \frac{\ln x}{\ln a}}$$

Solve for x: $\log(x+2)+\log(x+11)=1$

$$\log_{10}[(x+2)(x+11)]=1$$

$$10^{\log_{10}(x^2+13x+22)}=10^{1}$$

$$x^2+13x+22=10$$

$$x^2+13x+12=0$$

$$(x+12)(x+1)=0$$

$$x=\cancel{-12},\ \boxed{x=-1}$$

Test: $\cancel{\log(-10)+\log(-1)}$ $\quad\log(1)+\log(10)$ ✓

$$\underbrace{\infty}_{\infty+0} - \underbrace{\infty}_{\infty+0} \cancel{= 0}$$

Compute $\lim_{x\to\infty} \ln(e^x + e^{-x}) - \ln(2e^x + e^{-x})$

$$= \lim_{x\to\infty} \ln\left(\frac{e^x + e^{-x}}{2e^x + e^{-x}}\right)$$

ln is cts

$$= \ln\left(\lim_{x\to\infty} \frac{e^x + e^{-x}}{2e^x + e^{-x}}\right)$$

$$= \ln\left(\lim_{x\to\infty} \frac{\cancel{e^x}(1 + e^{-2x} \to 0)}{\cancel{e^x}(2 + e^{-2x} \to 0)}\right)$$

$$= \boxed{\ln\left(\tfrac{1}{2}\right) \overset{OR}{=} -\ln 2}$$

$(\overset{0}{\ln 1} - \ln 2$ OR $\ln(2^{-1}) = -(\ln 2)$