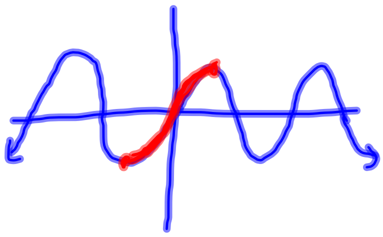
1 4.6: Inverse Trig Functions and Their Derivatives

sin, cos, tan and one-to-one functions:

Graph of $f(x) = \sin x$



NOT one-to-one

Restrict Domain

$f(x) = \sin x; \; x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

is one-to-one

What angle?

$y = \sin^{-1} x$ (or $\arcsin x$) if and only if $x = \sin y$; $y \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$y = \cos^{-1} x$ if and only if $x = \cos y$; $y \in [0, \pi]$

$y = \tan^{-1} x$ if and only if $x = \tan y$; $y \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Derivative of $y = \sin^{-1} x$:

means $x = \sin y$ Differentiate Implicitly

$1 = \cos y \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\cos(\sin^{-1} x)}$

Simplify using Reference Triangle

$\sin y = \frac{x}{1}$ Opp / Hyp

$\cos y = \frac{\sqrt{1-x^2}}{1}$ Adj / Hyp

$*\ \boxed{\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}}\ *$

$$\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$$

Examples:

Find the exact value of $\cos^{-1}\left(-\frac{1}{2}\right)$.

$$\boxed{\frac{2\pi}{3}}$$

(Thought process)

? $\cos y = -\frac{1}{2}$

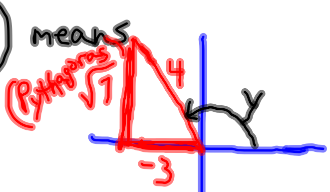
$\cos\frac{\pi}{3} = \frac{1}{2}$

Find the exact value of each of the following:

$$\tan\left(\underbrace{\cos^{-1}\left(-\frac{3}{4}\right)}_{y}\right)$$

$y = \cos^{-1}\left(\frac{-3}{4}\right)$ means

$\cos y = \frac{-3}{4}$



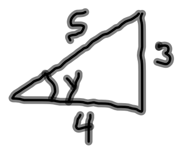
$$\tan y = \frac{\sqrt{7}}{-3} = \boxed{-\frac{\sqrt{7}}{3}}$$

$$\cancel{\sin}\left(2\,\underbrace{\cancel{\arcsin}\left(\frac{3}{5}\right)}_{y}\right) = \frac{6}{5} > 1$$

Not possible

$y = \arcsin\left(\frac{3}{5}\right)$ means

$\sin y = \frac{3}{5}$



$$\sin(2y) = 2\sin y\cos y$$
$$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$$
$$= \boxed{\frac{24}{25}}$$

cts

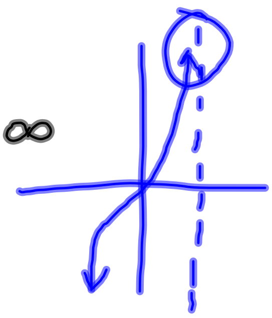
Compute $\lim_{x\to 0^-} \tan^{-1}\left(\frac{1-x^2}{2x^2-x}\right)$

$= \tan^{-1}\left(\lim_{x\to 0^-} \frac{1-x^2}{2x^2-x}\right)$ $\frac{1}{0}$ $\pm\infty$? $\frac{+}{+}$

$2x^2-x = x(2x-1)$, $-\ -$

$=$ "$\tan^{-1}(\infty)$"

$= \boxed{\frac{\pi}{2}}$ since as $x \to \frac{\pi}{2}^-$ $\tan x \to \infty$



Product Rule $(1-x^2)^{1/2}$ Chain Rule

On Your Own: Find the derivative of $f(x) = x \arcsin x + \sqrt{1-x^2}$

$$f'(x) = x \cdot \frac{1}{\sqrt{1-x^2}} + \arcsin x(1) + \frac{1}{\cancel{2}}(1-x^2)^{-1/2}(-\cancel{2}x)$$

$$= \cancel{\frac{x}{\sqrt{1-x^2}}} + \arcsin x - \cancel{\frac{x}{\sqrt{1-x^2}}}$$

$$= \arcsin x$$

Nov 4-12:36 PM