1 5.1: Graphical Interpretation of f, f', and f"

Graphical Interpretations of f':

If $f'(x) > 0$ for all $x \in (a, b)$ then f is increasing on (a,b)

If $f'(x) < 0$ for all $x \in (a, b)$ then f is decreasing on (a,b)

Example: Draw a function f from (1,0) to (4,5) with $f' > 0$:

Difference: "bend" in graphs

Definitions:

a differentiable function f is **concave up** on an interval (a,b) if and only if

f' is increasing on (a,b)

a differentiable function f is **concave down** on an interval (a,b) if and only if

f' is decreasing on (a,b)

Therefore...

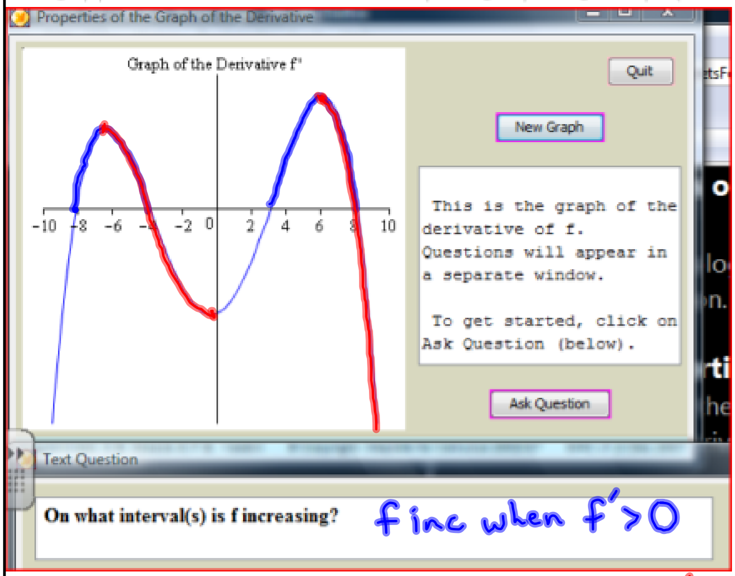
If $f''(x) > 0$ for all $x \in (a,b)$, then (f' inc) f is concave up on (a,b) + + $\cup$

If $f''(x) < 0$ for all $x \in (a,b)$, then (f' dec) f is concave down on (a,b) − − $\cap$

f has an <u>inflection point</u> at $x=a$ if f changes concavity at $x=a$. (f'' changes sign
f' changes direction)

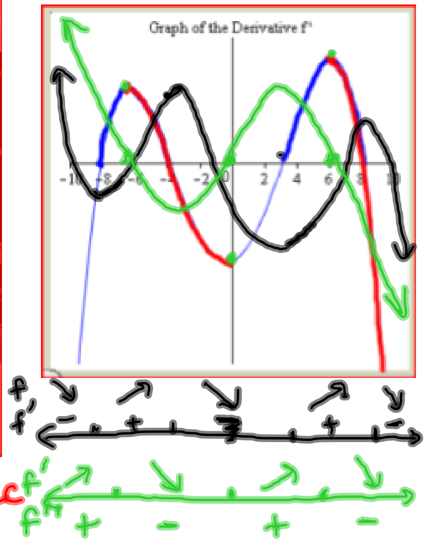
Examples:

Maplet: "Properties of the Graph of a Function/First Derivative/Second Derivative" found at http://calclab.math.tamu.edu/maple/maplets/ (Good source of On Your Own problems)



Graph of f

Graph of f''



On what interval(s) is f concave down? f conc down when f' dec

Sketch the graph of a continuous function which satisfies the following:

- $f'(x) < 0$ for $x \in (-1, 1)$
- $f'(x) > 0$ for $x \in (-\infty, -1) \cup (1, \infty)$
- $f(-1) = 4,\ f(1) = 0$
- $f''(x) < 0$ for all $x \neq 1$

f conc down