1 5.7: Antiderivatives

F is an antiderivative of f if and only if f is the derivative of F ($F' = f$)

Antiderivative Rules: NOTE: $f'(x) = x^2 \to f(x) = \frac{1}{3}x^3 + C$

Derivative $f'(x)$	Original Function $f(x)$	Derivative $f'(x)$	Original Function $f(x)$
x^n; $n \neq -1$	$\frac{1}{n+1}x^{n+1} + C$	$x'(t)\vec{i} + y'(t)\vec{j}$	$x(t)\vec{i} + y(t)\vec{j} + \vec{C}$
$f'(x) \pm g'(x)$	$f(x) \pm g(x) + C$	e^x	$e^x + C$
$cf'(x)$	$cf(x) + C$	$\frac{1}{x}$	$\ln\lvert x\rvert + C$
$\sin x$	$-\cos x + C$	$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x + C$
$\cos x$	$\sin x + C$	$\frac{1}{1+x^2}$	$\tan^{-1} x + C$
$\sec^2 x$	$\tan x + C$	$\frac{1}{x\sqrt{x^2-1}}$	$\sec^{-1} x + C$
$\sec x \tan x$	$\sec x + C$		
$-\csc^2 x$	$\cot x + C$		
$-\csc x \cot x$	$\csc x + C$		

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Examples:

Given $f'(x) = x^1 + x^2 + 4\sin x$:

a) Find $f(x)$

$$\boxed{f(x) = \frac{1}{2}x^2 + \frac{1}{3}x^3 - 4\cos x + C}$$

x y

b) Find $f(x)$ if $f(0) = 0$

$$f(x) = \frac{1}{2}x^2 + \frac{1}{3}x^3 - 4\cos x + C$$

$$0 = \frac{1}{2}\cdot 0^2 + \frac{1}{3}\cdot 0^3 - 4\cos 0 + C$$

$$0 = 0 + 0 - 4 + C \qquad C = 4$$

*NOTE: $C \neq$ initial value!

$$\boxed{f(x) = \frac{1}{2}x^2 + \frac{1}{3}x^3 - 4\cos x + 4}$$

The velocity of a particle along a straight line is given by $v(t) = e^t - \frac{1}{1+t^2} + t^2$. If the initial position is 4, find the position function at any time t.

$s(t) = e^t - \tan^{-1} t + \frac{1}{3}t^3 + C$ $\quad s(0) = 4$

$4 = e^0 - \tan^{-1} 0 + \frac{1}{3} \cdot 0^3 + C$

$4 = 1 - 0 + 0 + C \quad C = 3$

$\boxed{s(t) = e^t - \tan^{-1} t + \frac{1}{3}t^3 + 3}$

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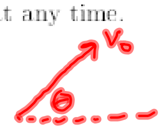
A projectile is fired from the top of a hill of height h kilometers with a speed of v_0 km/hr at an angle of θ. Neglecting air resistance, DERIVE formulas to find the position of the projectile at any time.



$$\vec{v}'(t) = \vec{a}(t) = -g\,\vec{j}$$

$$\vec{v}(t) = -gt\,\vec{j} + \vec{C}$$

$$\vec{v}(0) = v_0\cos\theta\,\vec{i} + v_0\sin\theta\,\vec{j}$$

$$\vec{v}(0) = (v_0\cos\theta)\vec{i} + (v_0\sin\theta)\vec{j} = -g(0)\vec{j} + \vec{C}$$

$$\vec{C} = (v_0\cos\theta)\vec{i} + (v_0\sin\theta)\vec{j}$$

$$\vec{r}'(t) = \vec{v}(t) = (v_0\cos\theta)\vec{i} + (-gt + v_0\sin\theta)\vec{j}$$

$$\vec{r}(t) = (v_0\cos\theta\, t + C_x)\vec{i} + \left(-\tfrac{1}{2}gt^2 + v_0\sin\theta\, t + C_y\right)\vec{j}$$

(Can place the constants inside each component)

$$\vec{r}(0) = 0\vec{i} + h\vec{j} = (v_0\cos\theta\cdot 0 + C_x)\vec{i} + \left(-\tfrac{1}{2}g\cdot 0^2 + v_0\sin\theta\cdot 0 + C_y\right)\vec{j}$$

$$\vec{r}(0) = 0\vec{i} + h\vec{j}$$

$C_x = 0$

$C_y = h$

$$\boxed{\vec{r}(t) = (v_0\cos\theta\, t)\vec{i} + \left(-\tfrac{1}{2}gt^2 + v_0\sin\theta\, t + h\right)\vec{j}}$$

On Your Own: x^{-2}

Given $f''(x) = \frac{1}{x^2} + e^x - 2$, $f'(1) = 0$, $f(1) = 1$, find $f(x)$.

$$f'(x) = -x^{-1} + e^x - 2x + C_1$$
$$= -\frac{1}{x} + e^x - 2x + C_1$$

$$0 = -\frac{1}{1} + e^1 - 2(1) + C_1$$
$$C_1 = 3 - e$$

$$f'(x) = -\frac{1}{x} + e^x - 2x + (3-e)$$
$$f(x) = -\ln|x| + e^x - x^2 + (3-e)x + C_2$$

$$1 = -\ln 1 + \cancel{e^1} - 1 + (3 - \cancel{e}) + C_2$$
$$C_2 = -1$$

$$\boxed{f(x) = -\ln|x| + e^x - x^2 + (3-e)x - 1}$$

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