1 6.1-6.2: Area

Introduction:

The second historical problem of Calculus involves finding the area under the graph of a function. Suppose we want to find the area under the graph of $f(x) = e^{-x^2}$ between $x = 0$ and $x = 2$ (denoted by $\int_0^2 e^{-x^2}\,dx$. We cannot find the exact area geometrically (or even using techniques you may have previously learned in Calculus!), so we approximate the area using rectangles (see figures in class):

What happens if more rectangles are used?

How many rectangles are needed to get the exact area?

Definitions:

partition:

Δx_i:

$||P||$:

x_i^*:

A **Riemann Sum**

The **area under the graph** of a positive function f from $x = a$ to $x = b$:

Examples:

Given $f(x) = x^2 + x$, write and compute a Riemann Sum to approximate the area under f from $x = 1$ to $x = 3$ using a partition $P = \{1, 2, 2.5, 3\}$. Let x_i^*= the midpoint of each subinterval.

Find the exact area under the graph of f from $x = 1$ to $x = 3$. (HINT: use n equally spaced partitions and take x_i^* =the right endpoint of each rectangle. Then let $n \to \infty$)