

Fall 2009 Math 151

LIFE Review for Final Exam

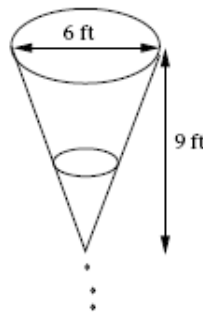
courtesy: David J. Manuel

(covering 1.1 - 6.4)

1. Compute $\frac{d}{dx} \left(\int_0^{\sin x} \sqrt{4+5t^4} dt \right)$

2. Compute $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \left(\frac{3i}{n} + 4 \right)^2$

3. A conical tank shown below is losing water at a rate of one cubic foot per minute. How fast is the height of the water changing when the height is 6 feet?



4. Suppose you have 600 grams of a radioactive substance which decays exponentially. After 5 years, $2/3$ of the substance remains. When will only 10% of the original substance remain?
5. A rectangular beam is cut from a cylindrical log of radius 10cm. The strength of the beam is proportional to the width and the square of the depth of a cross-section. Find the dimensions of the strongest beam.
6. Let \mathbf{a} be the vector $3\mathbf{i} + 2\mathbf{j}$ and \mathbf{b} be the vector $-\mathbf{i} + 4\mathbf{j}$. Find a unit vector in the direction of $2\mathbf{a} + 3\mathbf{b}$.
7. A triangle has vertices $A(-3, -2)$, $B(1, 2)$, and $C(0, 4)$. Find the angle of the triangle at vertex A .
8. Find parametric equations of the line tangent to the graph of $\mathbf{r}(t) = (t^3 - t)\mathbf{i} + \left(\frac{6t}{t+1} \right)\mathbf{j}$ at the point where $t = 1$.

9. Find a tangent vector of unit length to the curve $\mathbf{r}(t) = (t \sin t)\mathbf{i} + (t \cos t)\mathbf{j}$ at the point where $t = \pi$.

10. Find the slope-intercept equation of the line tangent to the curve $x = te^t$, $y = \frac{1}{\sqrt{t}}$ at the point $(e, 1)$.

11. Compute $\lim_{x \rightarrow 2} \frac{\sqrt{x+2} - \sqrt{2x}}{x^2 - 2x}$

12. Find the values of a and c which make the function below continuous and differentiable at $x = 3$:

$$f(x) = \begin{cases} ax^2 - 9x + c & \text{if } x < 3 \\ 2ax^2 + a^2x - 5 & \text{if } x \geq 3 \end{cases}$$

13. Compute $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 4}}{4x + 1}$.

14. Compute $\lim_{x \rightarrow \infty} \frac{\tan\left(\frac{1}{2x}\right)}{\frac{1}{3x}}$.

15. Given $f(x) = \sqrt{x^2 + 9}$, find $\lim_{h \rightarrow 0} \frac{f(4+h) - f(4)}{h}$.

16. Find the point(s) on the curve $xy^2 = 1$ where the tangent line also passes through the point $(0, 1)$.

17. The function $f(x) = e^{2x} + 2x^3 + 1$ is one-to-one. If $g = f^{-1}$, what is $g'(2)$?

18. Simplify $\tan(\sin^{-1} x)$

19. Given $f'(x) = \frac{3}{\sqrt{1-x^2}} + \frac{x}{3} + \frac{3}{x}$ and $f\left(\frac{1}{2}\right) = 0$, find $f(x)$.

20. Given $f(3) = 6$, $f'(3) = 0$, and $f''(3) = -4$. State everything you know about the graph of f at $x = 3$.

21. Find the linear approximation to $f(x) = \sqrt[3]{x}$ at $x = \frac{27}{8}$ and use it to approximate $\sqrt[3]{3}$.

22. Approximate the area under the graph of $f(x) = 16 - x^2$ between $x = -2$ and $x = 4$ using a partition $P = \{-2, 0, 4\}$ and taking x_i^* = midpoint.

23. Find the derivative of $g(x) = \ln(x^2 e^{4x}) + e^2 + x^{2e} + \pi^x$

24. Compute $\lim_{x \rightarrow 0} \frac{\ln(1+x) - x}{x^2}$.
25. Find f'' if $f(x) = e^{(-1/2)x^2}$.
26. Compute $\lim_{x \rightarrow 2^-} \frac{4x^2 + x}{x^2 - 4}$.
27. Find $\frac{d}{dx} \left(\frac{x}{\cos^2(4x)} \right)$.
28. Determine the interval(s) on which $f(x) = x^5 - 15x^4 + 6$ is decreasing and concave down.