Fall 2009 Math 151

Week in Review X

courtesy: David J. Manuel

(covering 4.5, 4.6, 4.8)

1 Section 4.5

1. Strontium-90 is a radioactive isotope with a half-life of 25 years. If there are 20 mg of Strontium present, how much remains after 15 years? When will there be only 5 mg left?

2. According to UN data, the world population at the beginning of 2000 was 6 billion and growing at a rate of 1.6%. Assuming an exponential growth model, estimate the world population at the beginning of 2010.

3. Newton's Law of Cooling states that the rate of change in the temperature of an object is proportional to the difference in temperature between the object and its surroundings. A cup of coffee with an initial temperature of $90°C$ is in a room that is held at a constant temperature of $20°C$. If the liquid cools to $75°C$ after 5 minutes, what will its temperature be after 15 minutes?

4. A tank initially contains 100kg of salt dissolved in 2000 liters of water. Pure water enters the tank at a rate of 50 L/min and the mixed solution is drained from the tank at the same rate. Find the amount of salt in the tank after t minutes.

2 Section 4.6

1. Find the exact values of each of the following:
   (a) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
   (b) $\arcsin(-1)$
   (c) $\cot\left(\sin^{-1}\left(\frac{4}{5}\right)\right)$
   (d) $\sin\left(2\arcsin\left(\frac{3}{5}\right)\right)$

2. Compute $\lim_{x\to 0}\tan^{-1}\left(\frac{1-x}{2x^2}\right)$

3. Use inverse functions to prove $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

4. Differentiate and simplify the following:
   (a) $f(x) = \arcsin\left(\frac{1}{x}\right)$
   (b) $y = x\tan^{-1}(2x) - \frac{1}{4}\ln(1+4x^2)$

3 Section 4.8

1. Compute the following limits:
   (a) $\lim_{t\to -2}\frac{t^3-t^2-t+10}{t^2+3t+2}$
   (b) $\lim_{x\to 0}\frac{1-\cos x}{2x^2}$
   (c) $\lim_{x\to 0^+} x\ln x$
   (d) $\lim_{x\to\infty}(1+e^{2x})^{1/x}$

2. The 4.3 WIR used the compound interest formula $A = P\left(1+\frac{r}{m}\right)^{mt}$. Find $\lim_{m\to\infty} A$