~~If~~

$f' > 0 \leftrightarrow f$ inc

$f' < 0 \leftrightarrow f$ dec

$++$ $--$

$\cup$ $\cap$

$f'' > 0 \leftrightarrow f'$ inc $\leftrightarrow f$ conc up

$f'' < 0 \leftrightarrow f'$ dec $\leftrightarrow f$ conc down

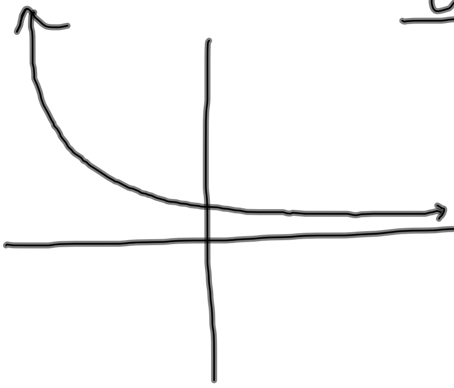
1 Section 5.1

1. True or False-there exists a function f such that $f(x) > 0$, $f'(x) < 0$ and $f''(x) > 0$ for all x. If true, sketch it; if false, explain why not.

$f > 0$ (above x-axis)

$f' < 0$ (f dec)

$f'' > 0$ (f conc up)



ONE example

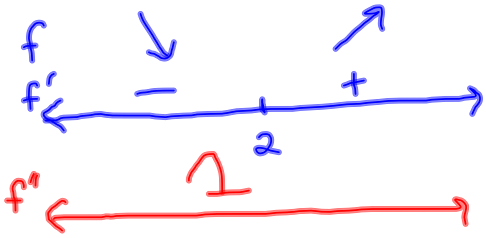
$f(x) = e^{-x} > 0$

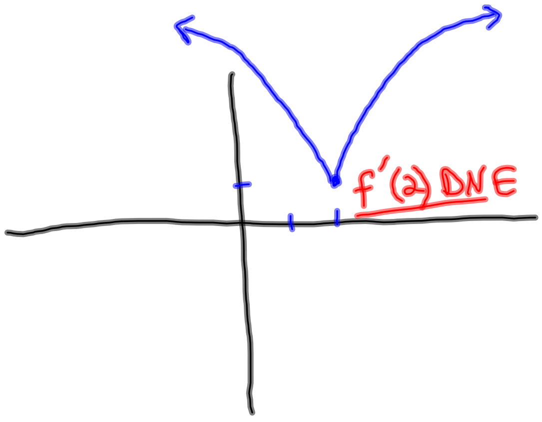
$f'(x) = -e^{-x} < 0$

$f''(x) = e^{-x} > 0$

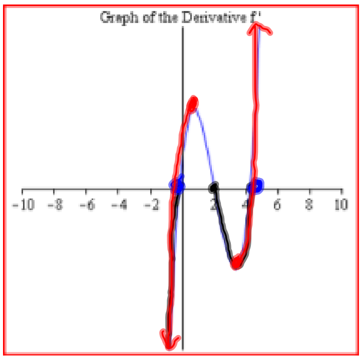
2. Sketch the graph of a function which satisfies the following:

- ✓ $f(2) = 1$ (2,1)
- $f'(x) < 0$ for $x < 2$ crit Value
- $f'(x) > 0$ for $x > 2$
- $f''(x) < 0$



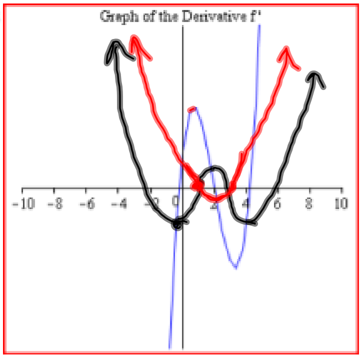


3. Maplets: "Properties of a Graph of a Function/First Derivative/Second Derivative" located at http://calclab.math.tamu.edu/maple/maplets



Graph of f

Graph of f''



f ↘ ↗ ↘ ↗

f' − + − +

0 2 4

f'' + − +

1 3

On what interval(s) is f decreasing? means $f' < 0$ (below x-axis)

Give the x-coordinate(s) of all points where f has a local minimum. $f' = 0$ (f' neg, then f' pos)

On what interval(s) is f concave up? f' inc

2 Section 5.2

1. Find the absolute maximum and absolute minimum of each of the following:

   $(6x-x^2)^{1/2}$

   (a) $f(x) = \sqrt{6x - x^2}$

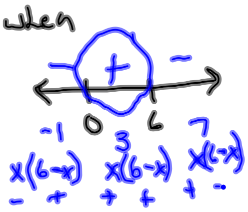
NOTE: Domain: $6x-x^2 \geq 0$

$[0,6]$

$6x-x^2 \geq 0$ when

$x(6-x)=0$

$x=0 \quad x=6$



Crit Vals

$$f'(x) = \frac{1}{2}(6x-x^2)^{-1/2}(6-2x) = 0$$

$$\frac{6-2x}{2\sqrt{6x-x^2}} = 0$$

$$6-2x=0$$

$$x=3$$

NOTE: $f'(0)$ $f'(6)$ DNE → 0, 6 are crit vals!

$f(0) = 0$

$f(3) = \sqrt{18-9} = 3$

$f(6) = 0$

$$\boxed{\begin{array}{l}\text{Abs max is } 3 \text{ when } x=3 \\ \text{Abs min is } 0 \text{ when } x=0,6\end{array}}$$

NOTE f must be cts on closed, bounded interval in order for this process to work.

$f(x) = \sqrt[3]{6x-x^2}$ has abs max $= \sqrt[3]{9}$ when $=3$

has NO abs min!!!

$(f(x) \to -\infty$ as $x \to \pm\infty)$

(b) $2\sec x - \tan x$ $\overset{\text{cts}}{\text{on}}$ the interval $\left[0, \frac{\pi}{4}\right]$ closed, bounded

crit vals

$$f'(x) = 2\sec x \tan x - \sec^2 x = 0$$

$$\sec x(2\tan x - \sec x) = 0$$

$$\cancel{\sec x = 0} \qquad 2\tan x - \sec x = 0$$

$$\frac{2\sin x}{\cos x} - \frac{1}{\cos x} = 0$$

$$2\sin x - 1 = 0$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}$$

Not in interval

~~f' DNE when $x = \frac{\pi}{2}$~~

$$f(0) = 2\overset{1}{\sec 0} - \overset{0}{\tan 0} = 2$$

$$f\left(\frac{\pi}{6}\right) = 2\overset{\frac{2}{\sqrt{3}}}{\sec\frac{\pi}{6}} - \overset{\frac{1}{\sqrt{3}}}{\tan\frac{\pi}{6}} = \frac{4}{\sqrt{3}} - \frac{1}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$f\left(\frac{\pi}{4}\right) = 2\overset{\sqrt{2}}{\sec\frac{\pi}{4}} - \overset{1}{\tan\frac{\pi}{4}} = 2\sqrt{2} - 1$$

$$2\sqrt{2} - 1 \overset{?}{<} 2$$

$$2\sqrt{2} \quad 3$$

$$8 \quad 9$$

$$\boxed{\begin{array}{l}\text{Abs max is } 2 \text{ when } x=0 \\ \text{Abs min is } \sqrt{3} \text{ when } x=\frac{\pi}{6}\end{array}}$$

(on calc)

cts closed, bdd

(c) $f(x) = x^2e^{-x}$ on the interval $[1, 4]$

crit vals

Product Rule

$$f'(x) = x^2(-e^{-x}) + (e^{-x})(2x) = 0$$

$$e^{-x}(-x^2 + 2x) = 0$$

$$xe^{-x}(-x+2) = 0$$

$$\cancel{x=0} \quad X \quad x=2$$

Not in interval

$$f(1) = 1e^{-1} = \frac{1}{e} = \frac{e^3}{e^4} \approx 27$$

$$f(2) = 4e^{-2} = \frac{4}{e^2} = \frac{4e^2}{e^4} \approx 36$$

$$f(4) = 16e^{-4} = \frac{16}{e^4} = \frac{16}{e^4} \approx 16$$

Abs max $4e^{-2}$ when $x=2$

Abs min $16e^{-4}$ when $x=4$

Not closed, bold interval

(d) $f(x) = \dfrac{\ln x}{x}$ on the interval $(0, \infty)$

Crit Vals

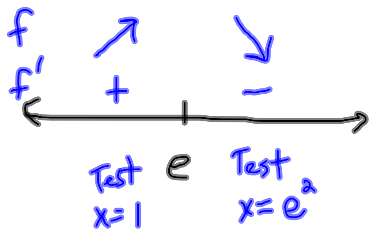
$$f'(x) = \frac{x \cdot \frac{1}{x} - \ln x(1)}{x^2} = 0$$

NOTE ~~$x=0$~~ is crit val
Not in interval

$$1 - \ln x = 0$$

$$\cancel{e^{\ln}}\, x = e^{1}$$

$$x = e$$



ONLY Critical value is rel max

∴ Absolute max is $\dfrac{\ln e}{e} = \dfrac{1}{e}$ when $x = e$
No absolute min

2. Find the critical values of $f(x) = 2x^3 - 15x^2 + 36x + 7$

$$f'(x) = 6x^2 - 30x + 36 = 0$$ (f' never undefined)

$$6(x^2 - 5x + 6) = 0$$

$$6(x-2)(x-3) = 0$$

$$\boxed{x=2 \quad x=3}$$

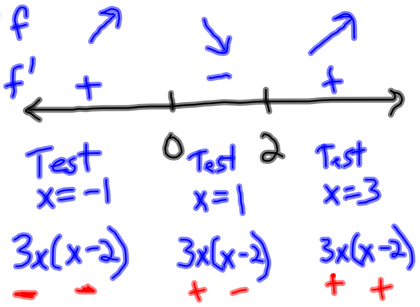
Nov 15-6:58 PM

1. Determine where the function $f(x) = x^3 - 3x^2 + 5$ is increasing, decreasing, concave up, and concave down.

Inc/Dec $f'(x) = 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x=0 \quad x=2$



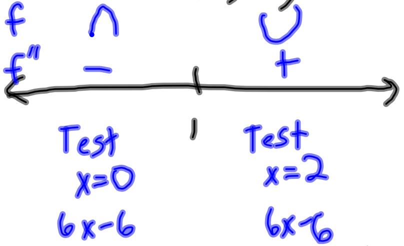
$$\boxed{\begin{array}{l} f \text{ inc } (-\infty,0)\cup(2,\infty) \\ f \text{ dec } (0,2) \end{array}}$$

NOTE rel max at (0,5)

rel min at (2,1)

Concavity $f''(x) = 6x - 6 = 0$

$x = 1$



$$\boxed{\begin{array}{l} f \text{ conc down } (-\infty,1) \\ f \text{ conc up } (1,\infty) \end{array}}$$

NOTE Point of Inflection at (1,3)

2. Find the horizontal and vertical asymptotes, intervals of direction, and intervals of concavity for $f(x) = \ln|1-x^2|$ and sketch the graph.

(4.3)

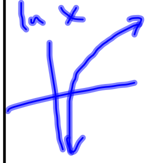
$f(x) = \ln|1-x^2|$

V.Asym $1-x^2=0$

$x = \pm 1$

No H Asym

ln x



$f'(x) = \frac{-2x}{1-x^2} = 0$

$x = 0, -1, 1$

f ↘ ↗ ↘ ↗

f' − + − +

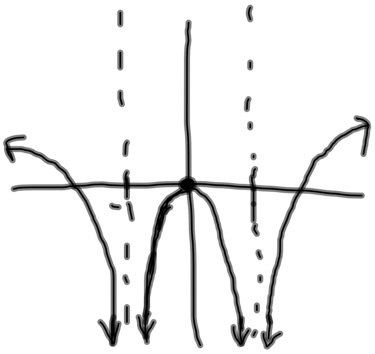
−1 0 1

Test $x=-2$ Test $x>-\frac{1}{2}$ Test $x=\frac{1}{2}$ Test $x=2$

$\frac{-2x}{1-x^2}$ + − $\frac{-2x}{1-x^2}$ + + − + − −

$$\boxed{\begin{array}{l} f \text{ inc } (-1,0)\cup(1,\infty) \\ f \text{ dec } (-\infty,-1)\cup(0,1) \end{array}}$$

NOTE: rel max at (0,0) NO rel min (asymp at ±1)



$f''(x) = \frac{(1-x^2)(-2)-(-2x)(-2x)}{(1-x^2)^2} = 0$

$= \frac{-2+2x^2-4x^2}{(1-x^2)^2}$

$= \frac{-2x^2-2}{(1-x^2)^2} = 0$ always − always +

f ∩ ∩ ∩

f'' − − −

−1 1

$\boxed{f \text{ conc down } (-\infty,-1)\cup(-1,1)\cup(1,\infty)}$

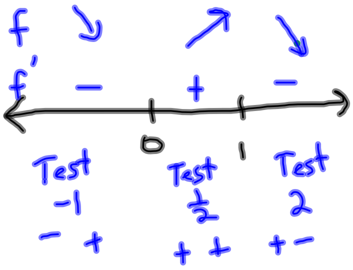
3. Determine where the function $f(x) = x^2e^{-2x}$ is increasing, decreasing, concave up, and concave down.

inc/dec

$$f'(x) = x^2\left(-2e^{-2x}\right) + e^{-2x}(2x) = 0$$

$$2x\,e^{-2x}(-x+1) = 0$$

$x=0$ ALWAYS + $x=1$



conc

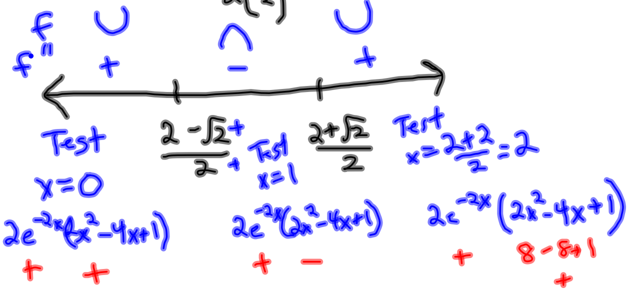
$$f'(x) = 2e^{-2x}(-x^2+x)$$

$$f''(x) = 2e^{-2x}(-2x+1) + (-x^2+x)(-4e^{-2x}) = 0$$

$$= 2e^{-2x}(-2x+1+2x^2-2x) = 0$$

$$= 2e^{-2x}(2x^2-4x+1) = 0$$

$$x = \frac{4 \pm \sqrt{(-4)^2-4(2)(1)}}{2(2)} = \frac{4\pm\sqrt{8}}{4} = \frac{2\pm\sqrt{2}}{2}$$



4. Find the inflection points of $f(x) = \underline{-x^2 \cos x} + \underline{6 \cos x} + \underline{4x \sin x}$, $x \in [-\pi, \pi]$.

Prod Rule

$$f'(x) = \underline{-x^2(-\sin x) + \cos x(-2x)} - \underline{6 \sin x} + \underline{4x \cos x + (\sin x)4}$$

$$= \underline{x^2 \sin x} + \underline{2x \cos x} - \underline{2 \sin x}$$

$$f''(x) = \underline{x^2 \cos x + \cancel{(\sin x)}(2x)} + \underline{\cancel{2x(-\sin x)} + \cancel{\cos x}(2)} - \underline{\cancel{2}\cos x}$$

$$= x^2 \cos x = 0$$

$\downarrow$ $x = 0$

$\downarrow$ $x = \frac{-\pi}{2}, \frac{\pi}{2}$

Infl pts at $x = \pm\frac{\pi}{2}$

(find y-values)