Fall 2009 Math 151

Week in Review XII

courtesy: David J. Manuel

(covering 5.5, 5.7, and 6.1)

1 Section 5.5

1. A farmer has 1200 feet of fencing to enclose a rectangular field bordered by a river on one side. If no fence is needed along the river, find the dimensions of the field which give the largest area.

2. A rectangular box with a square base and no top has volume V. Find the dimensions of the box which minimize surface area.

3. A right circular cylinder in inscribed in a right circular cone of radius $R$ and height $H$. Find the volume of the largest possible cylinder.

4. Find the shortest distance from the point $(0, 4)$ to the line $2x + 5y = -3$.

5. A movie screen is 24 feet tall and hangs 6 feet above eye-level. How far away from the screen should a person stand in order to maximize their viewing angle?

6. If a projectile is fired from the ground at an angle $\theta$ with initial speed $v_0$, the position of the projectile is given by $\mathbf{r}(t) = ((v_0 \cos \theta) t)i + ((-\frac{1}{2}gt^2 + (v_0 \sin \theta)t))j$. Find the angle which maximizes the horizontal range of the projectile.

2 Section 5.7

1. Find $f(x)$ for each of the following:

   (a) $f'(x) = x - \sqrt{x} + 10$

   (b) $f'(x) = \frac{1+x}{\sqrt{x}}$, $f(1) = 0$

   (c) $f'(x) = e^x - \frac{1}{x}$, $f(1) = 0$

   (d) $f''(x) = 1 + 2 \sin x - \cos x$, $f(0) = 3$, $f'(0) = 1$

   (e) $f'(x) = \sqrt{x} + \frac{2}{\sqrt{x}} + \frac{3}{\sqrt{1-x^2}}$

2. A force with magnitude 20 N acts in the positive $y$ direction on an object with a mass of 4 kg. The object starts at the origin with initial velocity $\mathbf{v}(0) = \mathbf{i} - \mathbf{j}$. Find its position function and speed at any time $t$.

3 Section 6.1

1. Compute $\sum_{i=1}^{5} i^2$.

2. Compute $\sum_{i=1}^{20} \left(2 - \left(\frac{1}{2}\right)^i\right)$.

3. Use the formulas on p367 of your text to compute $\lim_{n \to \infty} \frac{2}{n} \sum_{i=1}^{n} \left[9 - \left(1 + \frac{2i}{n}\right)^2\right]$. 