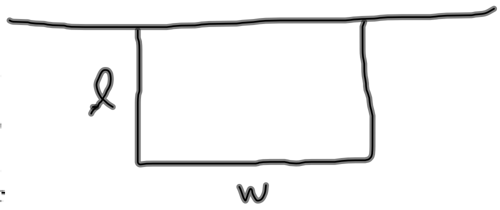
Reminder:

Night Before Drill Monday 30 Nov 7-9pm
in HELD 100

LIFE Review Thursday 10Dec (TBA)
in ZACH 102

1 Section 5.5

1. A farmer has 1200 feet of fencing to enclose a rectangular field bordered by a river on one side. If no fence is needed along the river, find the dimensions of the field which give the largest area.



Goal: Max $A = \ell w$

Restriction: $1200 = 2\ell + w$

$w = 1200 - 2\ell = 0$

Goal: Max $A = \ell(1200 - 2\ell)$; $\ell \in [0, 600]$

$A = 1200\ell - 2\ell^2$

$A' = 1200 - 4\ell = 0$

$\ell = 300$

Show max

3 options

① Extreme Value Thm

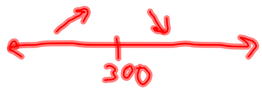
$A(0) = 0$

$A(300) = (300)(600) = 180{,}000$ MAX

$A(600) = 0$

OR

② First Der Test



OR

③ Second Der Test $A'' = -4$ ∩ Max

$\ell = 300$, $w = 1200 - 2(300) = 600$

$\boxed{300 \times 600 \text{ ft}}$

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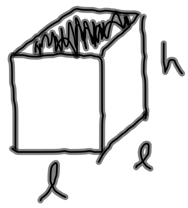
2. A rectangular box with a square base and no top has volume V~~.~~ constant Find the dimensions of the box which minimize surface area.



Goal: Min $S = l^2 + 4lh$

Restriction: $V = l^2 h$

$$h = \frac{V}{l^2}$$

Goal: Min $S = l^2 + 4\cancel{l}\left(\frac{V}{l^{\cancel{2}}}\right)$; $l > 0$

$$S = l^2 + \frac{4V}{l}$$

$$S' = \left(2l - \frac{4V}{l^2} = 0\right) l^2$$

$$2l^3 - 4V = 0$$

$$2l^3 = 4V$$

$$l = \sqrt[3]{2V}$$

Show min
(2nd Der Test)

$$S'' = 2 + \frac{8V}{l^3} > 0 \quad \cup \text{ Min}$$

$$l = \sqrt[3]{2V} \quad h = \frac{V}{\sqrt[3]{4V^2}} = \frac{\sqrt[3]{V}}{2}$$

$$\boxed{\sqrt[3]{2V} \times \sqrt[3]{2V} \times \frac{\sqrt[3]{V}}{2}}$$

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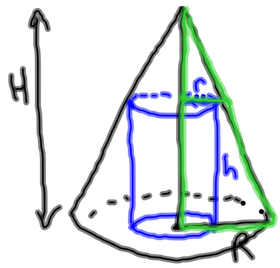
3. A right circular cylinder in inscribed in a right circular cone of radius R and height H. Find the volume of the largest possible cylinder.



Goal: Max $V = \pi r^2 h$

Restriction: $\frac{H-h}{r} = \frac{H}{R}$

$$RH - Rh = rH$$
$$RH - rH = Rh$$
$$h = \frac{RH - rH}{R}$$

Goal: Max $V = \pi r^2 \left(\frac{RH - rH}{R}\right)$; $r \in [0, R]$

$$= \frac{\pi}{R}\left(RHr^2 - r^3 H\right)$$

$$V' = \frac{\pi}{R}\left(2RHr - 3Hr^2\right) = 0$$

$$r(2RH - 3Hr) = 0$$

$$r = 0 \qquad r = \frac{2RH}{3H} = \frac{2R}{3}$$

Show max
Extr Val Thm

$$V(0) = 0$$
$$V\left(\tfrac{2R}{3}\right) = \pi\left(\frac{2R}{3}\right)^2\left(\frac{\cancel{R}H - \frac{2\cancel{R}}{3}H}{\cancel{R}}\right) = \boxed{\frac{4R^2H\pi}{27}} \text{ MAX}$$
$$V(R) = 0$$

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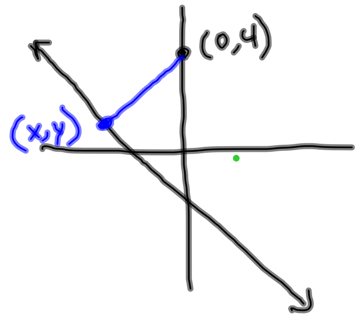
4. Find the shortest distance from the point $(0, 4)$ to the line $2x + 5y = -3$.



Goal: Min $d=\sqrt{(x-0)^2+(y-4)^2}$

Restriction: $2x+5y=-3$

$$x=-\frac{5}{2}y-\frac{3}{2}$$

Goal: Min $d=\sqrt{\left(-\frac{5}{2}y-\frac{3}{2}\right)^2+(y-4)^2}$ No domain restriction

easier to Min $d^2=D=\left(-\frac{5}{2}y-\frac{3}{2}\right)^2+(y-4)^2$

$$D'=\cancel{2}\left(-\frac{5}{2}y-\frac{3}{2}\right)\left(\frac{-5}{\cancel{2}}\right)+2(y-4)(1)=0$$

$$\left(+\frac{25}{2}y+\frac{15}{2}+2y-8\right)2$$

$$+25y+15+4y-16=0$$

$$29y-1=0$$

$$y=\frac{1}{29}$$

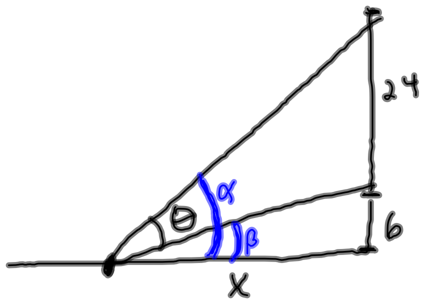
Show min (First Der Test)

$\searrow$ $\nearrow$ Min

$y=0$ $\quad \frac{1}{29}$ $\quad y=1$

Min $d=\boxed{\sqrt{\left(\frac{-5}{2}\cdot\frac{1}{29}-\frac{3}{2}\right)^2+\left(\frac{1}{29}-4\right)^2}}$ $\quad \frac{23}{\sqrt{29}}$

5. A movie screen is 24 feet tall and hangs 6 feet above eye-level. How far away from the screen should a person stand in order to maximize their viewing angle?



Goal: Max $\theta = \tan^{-1}\left(\frac{30}{x}\right) - \tan^{-1}\left(\frac{6}{x}\right)$; $x > 0$

$\tan\alpha = \frac{30}{x}$

$\tan\beta = \frac{6}{x}$

No restriction since no extra variables!

$$\theta' = \frac{1}{1+\left(\frac{30}{x}\right)^2} \cdot \frac{-30}{x^2} - \frac{1}{1+\left(\frac{6}{x}\right)^2} \cdot \frac{-6}{x^2}$$

$$= \frac{-30}{x^2+900} + \frac{6}{x^2+36} = 0$$

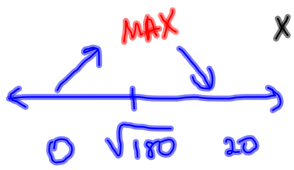
$$\frac{6}{x^2+36} \times \frac{30}{x^2+900}$$

$$6x^2 + 5400 = 30x^2 + 1080$$

$$4320 = 24x^2$$

$$x = \sqrt{\frac{4320}{24}} = \sqrt{\frac{1080}{6}} = \sqrt{180}$$

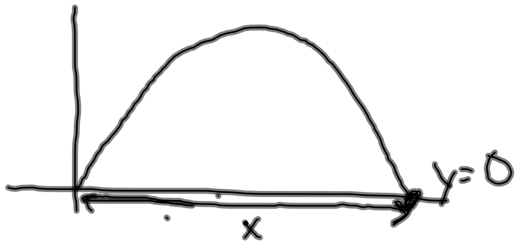
Show max
(First Der Test)



$\frac{-3\cancel{0}}{130\cancel{0}}$ $\frac{-6}{260}$ + $\frac{6}{436}$

$\boxed{\text{Stand } \sqrt{180} \text{ ft away}}$

6. If a projectile is fired from the ground at an angle θ with initial speed v_0 constant the position of the projectile is given by $\mathbf{r}(t) = ((v_0\cos\theta)\,t)\mathbf{i} + (-\frac{1}{2}gt^2 + (v_0\sin\theta)t)\mathbf{j}$. Find the angle which maximizes the horizontal range of the projectile.



Goal: Max $X = v_0\cos\theta\, t$

Restriction: $y = -\frac{1}{2}gt^2 + v_0\sin\theta\, t = 0$

$$t\left(-\frac{1}{2}gt + v_0\sin\theta\right) = 0$$

$t=0$ $\quad t = \frac{2v_0\sin\theta}{g}$

Goal: Max $X = v_0\cos\theta\left(\frac{2v_0\sin\theta}{g}\right)$

$$= \frac{2v_0^2}{g}\sin\theta\cos\theta \;;\quad \theta\in\left[0, \frac{\pi}{2}\right]$$

NOTE: $\frac{v_0^2}{g}(2\sin\theta\cos\theta)$

$\sin 2\theta$

Max is 1 when $\theta = 45^\circ$

$$x' = \frac{2v_0^2}{\cancel{g}}\left(\sin\theta(-\sin\theta) + \cos\theta(\cos\theta)\right) = 0$$

$\cos^2\theta - \sin^2\theta = 0 \rightarrow$ OR $\sin^2\theta = \cos^2\theta$

$\cos 2\theta = 0$ $\quad \tan^2\theta = 1$

$$\boxed{\theta = \frac{\pi}{4} = 45^\circ}$$

Show max $\quad x'' = -2\sin 2\theta \quad \cap$

2 Section 5.7

1. Find $f(x)$ for each of the following:

$x^{\frac{1}{4}}$

(a) $f'(x) = x^{1} - \sqrt[4]{x} + 10$

$$\boxed{f(x) = \frac{1}{2}x^{2} - \frac{4}{5}x^{\frac{5}{4}} + 10x + C}$$

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(b) $f'(x) = \dfrac{1+x}{\sqrt{x}}$, $f(\overset{x}{1}) = \overset{y}{0}$

$f'(x) = \dfrac{1}{\sqrt{x}} + \dfrac{x^1}{\sqrt{x}}$ $\left(x^{1-\frac{1}{2}}\right)$

$f'(x) = x^{-\frac{1}{2}} + x^{\frac{1}{2}}$

$f(x) = 2x^{\frac{1}{2}} + \frac{2}{3}x^{\frac{3}{2}} + C$

$0 = 2 \cdot 1^{1/2} + \frac{2}{3} \cdot 1^{3/2} + C \rightarrow C = -\frac{8}{3}$

$\boxed{f(x) = 2x^{1/2} + \frac{2}{3}x^{3/2} - \frac{8}{3}}$

$\cancel{\frac{1}{0x} - 1 + 1 = 0}$

(c) $f'(x) = e^x - \dfrac{1}{x}$, $f(\overset{x}{1}) = \overset{y}{0}$

$f(x) = e^x - \ln|x| + C$

$0 = e^1 - \ln 1 + C \qquad C = -e$

$$\boxed{f(x) = e^x - \ln|x| - e}$$

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(d) $f''(x) = 1 + 2\sin x - \cos x$, $f(0) = 3$, $f'(0) = 1$

x y x y

$f'(x) = x - 2\cos x - \sin x + C$ $\quad 1 = -2 + c$

$1 = 0 - 2\cos 0 - \sin 0 + C \rightarrow C = 3$

$f'(x) = x^1 - 2\cos x - \sin x + 3$

$f(x) = \frac{1}{2}x^2 - 2\sin x + \cos x + 3x + C$

$3 = \frac{1}{2}\cdot 0^2 - 2\sin 0 + \cos 0 + 3\cdot 0 + C \quad C = 2$

$$\boxed{f(x) = \frac{1}{2}x^2 - 2\sin x + \cos x + 3x + 2}$$

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$$x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} + 3\cdot\frac{1}{\sqrt{1-x^2}}$$ derivative of arcsin x

(e) $f'(x) = \sqrt{x} + \dfrac{2}{\sqrt{x}} + \dfrac{3}{\sqrt{1-x^2}}$

$$\boxed{f(x) = \frac{2}{3}x^{\frac{3}{2}} + 2\cdot 2x^{\frac{1}{2}} + 3\arcsin x + C}$$

2. A force with magnitude 20 N acts in the positive y direction on an object with a mass of 4 kg. The object starts at the origin with initial velocity $\mathbf{v}(0) = \mathbf{i} - \mathbf{j}$. Find its position function and speed at any time t.

$\vec{r}(0) = \vec{0}$

$\vec{F} = m\vec{a}$

$$\vec{F} = 20\vec{j} \rightarrow \vec{a}(t) = 5\vec{j}$$

$$\vec{v}(t) = 5t\vec{j} + \vec{C}$$

$$\vec{i} - \vec{j} = 5(0)\vec{j} + \vec{C} \qquad C = \vec{i} - \vec{j}$$

$$\vec{r}'(t) = \vec{v}(t) = 5t\vec{j} + (\vec{i} - \vec{j}) = \vec{i} + (5t - 1)\vec{j}$$

$$\vec{r}(t) = (t + C_x)\vec{i} + \left(\tfrac{5}{2}t^2 - t + C_y\right)\vec{j}$$

Can include C in each component

$$\vec{0} = (0 + C_x)\vec{i} + \left(\tfrac{5}{2}\cdot 0^2 - 0 + C_y\right)\vec{j}$$

$$C_x = 0$$

$$C_y = 0$$

$$\boxed{\vec{r}(t) = t\vec{i} + \left(\tfrac{5}{2}t^2 - t\right)\vec{j}}$$

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1. Compute $\sum_{i=1}^{5} i^2$. $= 1^2 + 2^2 + 3^2 + 4^2 + 5^2$

$= \boxed{59}$

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2. Compute $\displaystyle\sum_{i=1}^{20}\left(2-\left(\frac{1}{2}\right)^{i}\right)$

$$= \sum_{i=1}^{20} 2 - \sum_{i=1}^{20}\left(\frac{1}{2}\right)^{i}$$

$$\underbrace{\left(\frac{1}{2}\right)}_{a}\underbrace{\left(\frac{1}{2}\right)}_{r}^{i-1}$$

Geometric Series

$$\sum_{i=1}^{n} ar^{i-1} = \frac{a-ar^{n}}{1-r}$$

$$= 2(20) - \frac{\frac{1}{2}-\frac{1}{2}\left(\frac{1}{2}\right)^{20}}{1-\frac{1}{2}}$$

$$= \boxed{40 - \left(1-\left(\frac{1}{2}\right)^{20}\right)}$$

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3. Use the formulas on p367 of your text to compute $\lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n}\left[9-\left(1+\frac{2i}{n}\right)^2\right]$..

$$= \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n}\left[9-\left(1+\frac{4i}{n}+\frac{4i^2}{n^2}\right)\right]$$

$$= \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n}\left(8-\frac{4i}{n}-\frac{4i^2}{n^2}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n}\left(\sum_{i=1}^{n}8-\sum_{i=1}^{n}\frac{4i}{n}-\sum_{i=1}^{n}\frac{4i^2}{n^2}\right)$$

$$= \lim_{n\to\infty}\left[\frac{2}{n}\sum_{i=1}^{n}8-\frac{2}{n}\sum_{i=1}^{n}\frac{4i}{n}-\frac{2}{n}\sum_{i=1}^{n}\frac{4i^2}{n^2}\right]$$

$$= \lim_{n\to\infty}\left[\frac{2}{n}\sum_{i=1}^{n}8-\frac{8}{n^2}\sum_{i=1}^{n}i-\frac{8}{n^3}\sum_{i=1}^{n}i^2\right]$$

$$= \lim_{n\to\infty}\left[\frac{2}{\cancel{n}}\cdot 8\cancel{n}-\frac{\overset{4}{\cancel{8}}}{\cancel{n^2}}\cdot\overset{\text{GIVEN}}{\frac{\cancel{n}(n+1)}{\cancel{2}_1}}-\frac{\cancel{8}^{4}}{n^{\cancel{3}2}}\cdot\overset{\text{GIVEN}}{\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}\,3}}\right]$$

$$= \lim_{n\to\infty}\left(16-\frac{4(n+1)}{n}-\frac{4(2n^2+3n+1)}{3n^2}\right)$$

$$= 16-4-\frac{8}{3}=\boxed{\frac{28}{3}}$$