1. Find the domain of the function \( f(x) = \sqrt{2x - 3} \).

All \( x \) such that \( 2x - 3 \geq 0 \)

\[
2x \geq 3 \\
x \geq \frac{3}{2} \quad \text{or} \quad \left[ \frac{3}{2}, \infty \right)
\]

(NOTE): \( \sqrt{x^2 + 2x - 3} \)

Critical Values

\[
x^2 + 2x - 3 = 0 \\
(x + 3)(x - 1) = 0 \\
x = -3, x = 1
\]

\( \sqrt{2x - 3} \) \(-\infty, \infty\)

\(-\infty, -3\] \cup \([1, \infty)\)
2. Find the domain of the function \( g(x) = \frac{x + 6}{x^2 - 3x - 4} \).

All \( x \) except

\( x^2 - 3x - 4 = 0 \)
\( (x-4)(x+1)=0 \)
\( x= 4, \ x= -1 \)

Domain: all \( x \neq 4, -1 \) or \( \{ x | x \neq -1, 4 \} \)
\( (-\infty, -1) \cup (-1, 4) \cup (4, \infty) \)
3. Let \( g(t) = t^2 - t + 3 \). Compute and expand \( g(t + h) - g(t) \).

\[
g(t+h) - g(t) = [(t+h)^2 - (t+h) + 3] - [t^2 - t + 3]
= t^2 + 2th + h^2 - t - h + 3 - t^2 + t - 3
= 2th + h^2 - h
\]

*NOT* \( t^2 - t + 3 + h \)
4. Let \( f(x) = \sqrt{x + 1} \) and \( g(x) = x^2 - 2 \). Compute \( f(g(x)) \) and \( g(f(x)) \) and state the domain of each.

\[
f(g(x)) = f(x^2 - 2)
\]

\[
= \frac{\sqrt{(x^2 - 2) + 1}}{\sqrt{x^2 - 1}}
\]

Domain: \( x^2 - 1 > 0 \)
C.V. \( x^2 - 1 = 0 \)
\( (x+1)(x-1) = 0 \)
\( x = -1, 1 \)

Test \( x = 2 \) \( -1 < 0 \) \( 1 > 2 \)

\[
g(f(x)) = g(\sqrt{x+1})
\]

\[
= (\sqrt{x+1})^2 - 2
\]

No restriction on answer, but restriction on \( f \) inside
\( x + 1 \geq 0 \)
\( x \geq -1 \)

\( (-1, \infty) \)
5. A box with a square base and no top has a volume of 40 cubic centimeters. Write the surface area of the box as a function of:

a) its width
b) its height

\[ SA = w^2 + 4wh \]
\[ V = w^2h = 40 \]

a) Solve for \( h \):
\[ h = \frac{40}{w^2} \]
\[ S = w^2 + 4w \cdot \frac{40}{w^2} = \sqrt{w^2 + \frac{160}{w}} \]

b) Solve for \( w \):
\[ w = \frac{40}{h} \]
\[ w = \sqrt{\frac{40}{h}} \]
\[ S = \frac{160}{h} + 4 \sqrt{\frac{10}{h}} \cdot h = \frac{40}{h} + 4 \sqrt{40h} \]
1. The graph of \( f(x) = ax^2 + bx + c \) is shown below. Find \( a, b, \) and \( c. \)

\[
f(x) = \frac{1}{2}(x-2)^2 + 1 \\
= \frac{1}{2}(x^2 - 4x + 4) + 1 \\
= \frac{1}{2}x^2 - 2x + 2 + 1 \\
= \frac{1}{2}x^2 - 2x + 3 \\
a \quad b \quad c
\]

Graph of \( x^2 \) shifted:
- \( a \) 2 right
- \( b \) 1 up

New graph:
\[
y = a(x-2)^2 + 1 \\
3 = a(0-2)^2 + 1 \\
3 = 4a + 1 \\
3 = 4a \\
a = \frac{1}{4}
\]
2. “Shifting Functions” Maplet located at http://calclab.math.tamu.edu/maple/maplets/ (only works on OAL machine, Calclab machine, or any machine with Maple installed on it)
1. Let \( \theta \) be an angle in a right triangle. If \( \sin \theta = \frac{1}{3} \), find \( \cot \theta \).

\[
\cot \theta = \frac{\sqrt{8}}{1} \quad \text{by Pythagorean}
\]
2. Let $\theta$ be an angle in a right triangle. If $
abla \sin^2 \theta = \frac{1}{3}$, find $\tan^2 \theta$.

\[
\tan^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{\sin^2 \theta}{1 - \sin^2 \theta} = \frac{1/3}{1 - 1/3} = \frac{1}{2}
\]
3. Given $\cos \theta = \frac{2\text{ Adj}}{5}$ and $\sin \theta < 0$, find $\tan \theta$.

$$\tan \theta = \frac{\text{ Opp}}{\text{ Adj}} = \frac{-\sqrt{21}}{2}$$
4. Sketch the graph of \( f(x) = -3 \sin \left( \frac{\pi}{3} x \right) \) over two periods starting at \( x = 0 \).
5. The graph of \( f(x) = A \cos(bx) \) is shown below. Find \( A \) and \( b \).

\[ \cos: \quad \begin{array}{c}
A = -2 \\
\text{period} = \frac{4}{1} = \frac{2\pi}{b} \\
b = \frac{\pi}{2}
\end{array} \]
6. Use identities to write $\sin^5 x$ in the form $\sin x P(\cos x)$, where $P$ is a polynomial function.

\[
\sin^5 x = \sin x (\sin^4 x) = \sin x (\sin^2 x)^2 = \sin x (1-\cos^2 x)^2 = \frac{(1-\cos x)(1-\cos^2 x)}{\sin x(1-\cos^2 x + \cos^4 x)}
\]
7. Find all values of $x$ between 0 and $2\pi$ for which $\sin x = \sin 2x$.

\[
\begin{align*}
\sin x - \sin 2x &= 0 \\
\sin x - 2\sin x \cos x &= 0 \\
\sin x (1 - 2\cos x) &= 0
\end{align*}
\]

\[
\begin{align*}
\sin x &= 0 \\
\cos x &= \frac{1}{2}
\end{align*}
\]

$x = 0, \pi, 2\pi, \frac{\pi}{3}, \frac{5\pi}{3}$
1. Find a vector which represents the directed line segment from $A(3,5)$ to $B(-4,-1)$. Sketch $\overline{AB}$ and the vector in its standard position.

$\vec{c} = (-4\hat{i} - \hat{j}) - (3\hat{i} + 5\hat{j})$

$\vec{c} = (-4 - 3)\hat{i} + (-1 - 5)\hat{j}$

$\vec{c} = \begin{pmatrix} -7 \\ -6 \end{pmatrix}$
2. Find a unit vector parallel to the vector from the point $(5, 2)$ to the point $(-1, 3)$.

\[
\vec{a} = (-1 - 5\hat{i} + 2\hat{j}) = (-6\hat{i} + 2\hat{j})
\]

\[
\text{End} - \text{Start} \uparrow
\]

\[
\hat{n}_a = \frac{1}{\sqrt{37}} (-6\hat{i} + 2\hat{j})
\]

\[
|\vec{a}| = \sqrt{(-6)^2 + 2^2} = \sqrt{37}
\]

\[
\langle \frac{-6}{\sqrt{37}}, \frac{2}{\sqrt{37}} \rangle
\]
3. Given \( \mathbf{a} = \langle 5, 2 \rangle \) and \( \mathbf{b} = -2i + 4j \), find \( \mathbf{a} - \mathbf{b} \) and \( 3\mathbf{a} + 4\mathbf{b} \).

\[
\mathbf{a} - \mathbf{b} = (5\hat{i} + 2\hat{j}) - (-2\hat{i} + 4\hat{j}) \\
= (5 + 2)\hat{i} + (2 - 4)\hat{j} \\
= 7\hat{i} - 2\hat{j}
\]

\[
3\mathbf{a} + 4\mathbf{b} = 3(5\hat{i} + 2\hat{j}) + 4(-2\hat{i} + 4\hat{j}) \\
= (15\hat{i} + 6\hat{j}) + (-8\hat{i} + 16\hat{j}) \\
= 7\hat{i} + 22\hat{j}
\]
4. Two people are to pull ropes attached to a 50 kg box (on a frictionless surface). The rope to the left of the box makes an 60° angle with the box and is pulled with a force of 6 N. The rope to the right makes a 45° angle with the box and is pulled with a force of 10 N. Determine how fast the box accelerates and at what angle from the vertical.

\[
\begin{align*}
\vec{F}_1 &= (6 \cos 60°) \hat{i} + (6 \sin 60°) \hat{j} \\
    &= -3 \hat{i} + 3 \sqrt{3} \hat{j} \\
\vec{F}_2 &= (10 \cos 45°) \hat{i} + (10 \sin 45°) \hat{j} \\
    &= 5 \sqrt{2} \hat{i} + 5 \sqrt{2} \hat{j} \\
\vec{F} &= (-3 + 5 \sqrt{2}) \hat{i} + (3 \sqrt{3} + 5 \sqrt{2}) \hat{j} \\
|\vec{F}| &= \sqrt{(-3 + 5 \sqrt{2})^2 + (3 \sqrt{3} + 5 \sqrt{2})^2} \\
\tan \theta &= \frac{y}{x} = \frac{3 \sqrt{3} + 5 \sqrt{2}}{-3 + 5 \sqrt{2}} \\
\text{accel} &= \frac{|\vec{F}|}{50} \\
\text{answer} &= 90° - \tan^{-1}\left(\frac{3 \sqrt{3} + 5 \sqrt{2}}{-3 + 5 \sqrt{2}}\right)
\end{align*}
\]
5. (See Figure 11 on p52 for diagram). A 50-lb weight hangs from two wires. Find the magnitudes of the tensions in each wire.

\[ \sum F = 0 \]

\[ \vec{G} = -50 \hat{j} \]

Let \( A = |T_1| \) and \( B = |T_2| \)

\[ \vec{T}_1 = (-A \cos 50^\circ) \hat{i} + (A \sin 50^\circ) \hat{j} \]
\[ \vec{T}_2 = (B \cos 32^\circ) \hat{i} + (B \sin 32^\circ) \hat{j} \]

\[ \begin{align*}
\vec{i} \text{ comp} & \quad -A \cos 50^\circ + B \cos 32^\circ = 0 \\
\vec{j} \text{ comp} & \quad A \sin 50^\circ + B \sin 32^\circ - 50 = 0 \\
\end{align*} \] 

Solve for \( A, B \)