Fall 2009 Math 151

Night Before Drill for Exam I

courtesy: David J. Manuel

(covering 0.1-3.2 & App D)

1 Section 0.1

1. Given $f(x) = \dfrac{x}{x+1}$, find and simplify $f(f(x))$ and state its domain.

2. A tank contains 2000 liters of pure water. A brine solution containing 20 grams of salt per liter of water is pumped into the tank at a rate of 40 liters per minute. Write a function $C(t)$ which represents the concentration of the solution, in grams per liter, after t minutes.

3. A box with a square base and no top is to have a volume of 200 cubic centimeters. Find a formula for the surface area of the box as a function of the length of one side of the square.

2 Appendix D

1. Convert 240° to radians and find the exact trigonometric ratios for this angle.

2. Solve for x: $\cos 2x - \sin x = 0$.

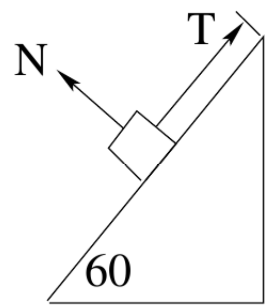
3. If $\sin x = \dfrac{3}{4}$ and $0 < x < \dfrac{\pi}{2}$, find all other trigonometric ratios of x.

3 Section 1.1

1. Find a unit vector which points in the same direction as the vector from the point $(-1, 5)$ to the point $(2, 3)$.

2. A woman walks due west on a ship at a rate of 4 miles per hour. The ship is moving northeast ($N45°E$) at a speed of 20 miles per hour. Find the direction and speed of the woman relative to the water.

3. A box is held in place by a cable on a friction-free ramp as shown below. If the mass of the box is 50kg, find the magnitude of the tension in the cable.



4 Section 1.2

1. Find the area of the triangle whose vertices are at the points $A(-1, 2)$, $B(2, 1)$ and $C(0, 5)$. (NOTE: $A = \frac{1}{2}bh$ or $A = \frac{1}{2}ab\sin\theta$)

2. Let $\mathbf{a} = -2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = < 1, 2 >$. Find the scalar and vector projection of $\mathbf{a}$ onto $\mathbf{b}$.

3. A 10 kg suitcase sits atop the ramp of a cruise ship. The ramp is 4 meters tall and is attached 2 meters (horizontally) away from the dock. Assuming no friction, find the work done by gravity in sliding the suitcase from the top of the ramp to the bottom.

5 Section 1.3

1. Find parametric equations of the line which passes through the points $(-3, -1)$ and $(-1, 5)$.

2. Find a Cartesian equation of the curve parametrized by $x = \cos t$, $y = \cos 2t$ and sketch the graph.

6 Section 2.2-2.3

1. Compute the following limits:

   (a) $\lim_{x\to 5} \frac{2x^2-13x+15}{x^2-3x-10}$

   (b) $\lim_{t\to 3} \frac{\frac{1}{t}-\frac{1}{3}}{t-3}\mathbf{i}+(2t-3)\mathbf{j}$

   (c) $\lim_{x\to 0} x^2\cos\left(\frac{1}{x^2}\right)+5$

   (d) $\lim_{x\to 2^+} \frac{2x}{4-x^2}$

7 Section 2.5

1. Determine the values of x for which the function below is not continuous. Explain your answers.

$$f(x)=\begin{cases} x+2 & \text{if } x\le -1 \\ \frac{|x-1|}{x-1} & \text{if } -1<x<1 \\ 0 & \text{if } x=1 \\ -x^2 & \text{if } 1<x<3 \\ -2x-3 & \text{if } x\ge 3 \end{cases}$$

2. Let $f(x) = \begin{cases} x-c & \text{if } x<3 \\ 3c-x & \text{if } x\ge 3 \end{cases}$ Find the value of c that makes f continuous at $x=3$.

3. Show that the equation $x^3-2x^2+x=5$ has a solution.

8 Section 2.6

1. Compute $\lim_{x\to\infty} \frac{4x^2+3x+5}{-2-x+5x^2}$

2. Compute $\lim_{x\to\infty} \sqrt{x^2+3x+1}-x$.

3. Compute $\lim_{x\to-\infty} \frac{\sqrt{4x^2+x-1}}{5x-3}$.

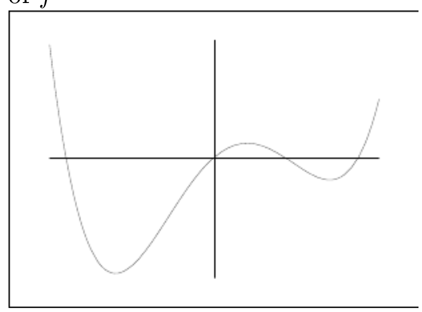
9 Section 2.7, 3.1

1. Given $\mathbf{r}(t)=(3t)\mathbf{i}+(4t-t^2)\mathbf{j}$, use the limit definition to find a vector tangent to the graph at the point where $t=1$.

2. Use the limit definition to find the derivative of $f(x)=\sqrt{2x-3}$

3. Given the graph of f passes through the point $(-1,4)$ and the equation of the line tangent to f at this point is $y=5x+9$, compute $\lim_{x\to-1} \frac{f(x)-4}{x+1}$.

4. Given the graph of f below, sketch the graph of f'



10 Section 3.2

1. Find the derivative of $f(x)=3x-2\sqrt{x}+\frac{1}{\sqrt{x}}$.

2. Given $f(3)=-2$ and $f'(3)=4$, find $g'(3)$ if $g(x)=x^2f(x)$

3. Given $f(x)=\frac{x^3+1}{x^2+1}$, find the equation of the tangent line at the point where $x=-1$.

4. Find the points on the graph of $y=x^2+x$ where the tangent line also passes through the point $(2,-10)$.