Fall 2009 Math 151

Night Before Drill for Exam II

courtesy: David J. Manuel

(covering 3.3 - 4.2)

1 Section 3.3

1. A particle is moving along a straight line with position equation $s = f(t) = t^2 - 2t + 3$, where t is in seconds and s is in feet.
   (a) When is the particle at rest?
   (b) Find the distance traveled by the particle during the first 4 seconds.
2. A tank contains 2000 liters of pure water. A brine solution containing 20 grams of salt per liter of water is pumped into the tank at a rate of 40 liters per minute. Find the rate of change in the concentration of the solution after 10 minutes.

2 Section 3.4

1. Compute $\lim_{x \to 0} \dfrac{3\cos x - 3 + 4\sin x}{5x}$
2. Compute $\lim_{x \to 0} \dfrac{\sin^2(8x)}{9x^2}$.
3. Find all values of x between 0 and 2π where the tangent line to $f(x) = 2x - \tan x$ is horizontal.

3 Section 3.5

1. Given $f(x) = \sin^4 x$, compute $f'\left(\dfrac{\pi}{3}\right)$.
2. Find the derivative of $f(x) = \cos\sqrt{x^2+1}$.
3. Suppose f is a differentiable function such that $f(1) = 6$, $f'(1) = -2$, $f(3) = 6$, $f'(3) = 4$, $f(6) = 5$, $f'(6) = -1$, $f(9)$, and $f'(9) = 1$.
   (a) If $g(x) = (f(x))^3$,find $g'(3)$
   (b) If $h(x) = f(f(x))$, find $h'(3)$

4 Section 3.6

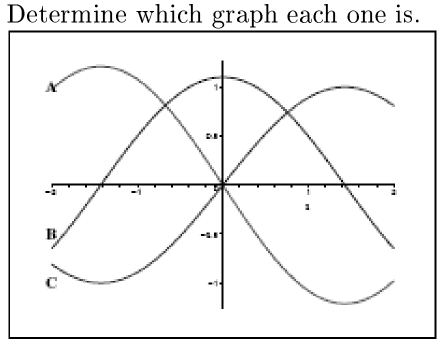
1. Find the slope of the line tangent to the curve given by $-xy^2 + 3y^3 = -3$ at the point $(0, -1)$.
2. Find $\dfrac{dy}{dx}$ given $y^2 + xy = 8$.
3. Show that the curves $x^2 + y^2 = 2y$ and $x^2 + y^2 = 4x$ are orthogonal.

5 Section 3.7

1. The position of a particle is given by $\mathbf{r}(t) = (e^{-2t}\cos(2t))\mathbf{i} + (e^{-2t}\sin(2t))\mathbf{j}$. Find the velocity and speed of the particle when $t = 0$.
2. Given the curve $\mathbf{r}(t) = (6\sin t)\mathbf{i} + (3\cos t)\mathbf{j}$, find a unit tangent vector at the point where $t = \dfrac{\pi}{3}$.

6 Section 3.8

1. Let $f(x) = (1+x^2)^{3/2}$. Find $f''(0)$.
2. Find the 20th derivative $f^{20}(x)$ of $f(x) = xe^x$.
3. The graphs of f, f',and f'' are shown below. Determine which graph each one is.



7 Section 3.9

1. Given the curve parametrized by $x = t^2$, $y = t^3 - 3t$, find the equation of the tangent line at the point $(4, -2)$.
2. Find the points on the curve $x = t^3 - 3t^2 - 9t + 1$, $y = t^3 + 3t^2 - 9t + 1$ at which the tangent line is horizontal or vertical (indicate which).

8 Section 3.10

1. A man 6 feet tall is walking at the rate of 3 ft/sec away from a ground-level spotlight toward a wall which is 18 feet away. How fast is his shadow on the wall changing when he is 10 feet from the spotlight?

2. Fifty meters from a launching pad, a man observes a helicopter taking off. The helicopter lifts off vertically and rises at a speed of 52 m/sec. At the instant its height is 120m, how fast is the distance between the helicopter and the man changing?

3. Sand is being dumped into a tank in the shape of a cone whose radius is 6 meters and height is 8 meters. If the sand is pouring in at a rate of 100 cm^3 per second, how fast is the height of the sandpile increasing when the height is 75 cm? The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

9 Section 3.11

1. Use differentials or linear approximation to estimate $\sqrt[3]{8.012}$.

2. Find the linear approximation to $f(x) = \sqrt{1+2x}$ near $x = 0$.

3. Find the quadratic approximation to $f(x) = \cos x$ near $x = \frac{\pi}{6}$.

10 Section 4.1

1. Compute $\lim_{x\to 0^-} \frac{2}{1+e^{\cot x}}$.

2. Find the first and second derivatives of $f(x) = e^{x^3+x}$

3. Show that the function $y = Cxe^{-2x}$ (C is a constant) is a solution to the differential equation $y'' + 4y' + 4y = 0$.

11 Section 4.2

1. The function $f(x) = \frac{4x+3}{2x+1}$ is one-to-one. Let $g = f^{-1}$. Find $g'(1)$.

2. Given f is a one-to-one function with $f(4) = 2$, $f'(4) = -3$, $f(2) = 6$, $f'(2) = -5$, $f(6) = 4$, and $f'(6) = -1$. If $g = f^{-1}$, find $g'(2)$.

3. The function $f(x) = x^3 - 2x^2 + 5x$ is one-to-one. Let $g = f^{-1}$. Find $g'(10)$.