

1 Ch 0, App D: Review

a rule that connects elements such that each $x \in D$ connects to at most one $y \in R$

Functions: $f(M) = A$

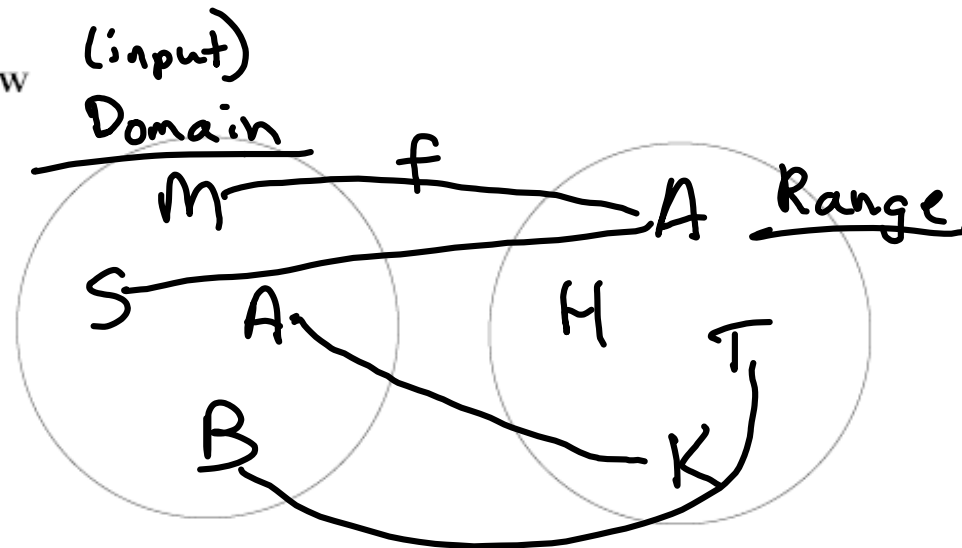
Graph of a Function:

Set of all points (x, y) such that $y = f(x)$

(i.e., $x \in D$ connects to $y \in R$)

Combining Functions: $(f \overset{\pm}{\underset{\div}{\cdot}} g)(x) = f(x) \overset{\pm}{\underset{\div}{\cdot}} g(x)$

Composition: $(f \circ g)(x) = f(g(x))$ output of g becomes input of f



Examples: denom $\neq 0$ or $\sqrt{\text{nonnegative}}$

Find the domain and ~~range~~ of $f(x) = \sqrt{24 + 2x - x^2}$

We need $24 + 2x - x^2 \geq 0$

Critical Values

$$-x^2 + 2x + 24 = 0$$

$$-(x^2 - 2x - 24) = 0$$

$$\textcircled{-} (x-6)(x+4) = 0$$

$$x=6, x=-4$$

(On Your Own): Find the domain of $f(x) = \frac{x}{\sqrt{|x-2|-1}}$

We need $|x-2|-1 > 0$

$$|x-2| > 1$$

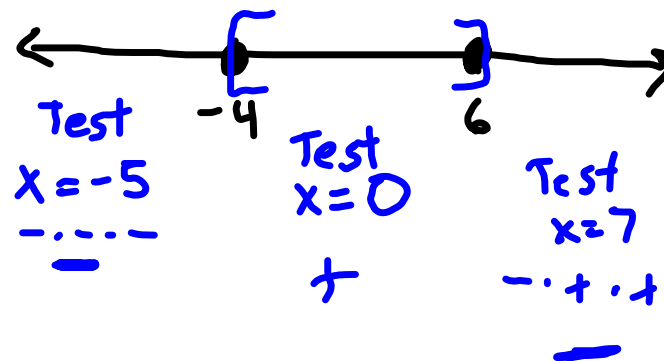
$$x-2 > 1 \text{ or } -(x-2) > 1$$

$$x > 3$$

$$x-2 < -1$$

$$x < 1$$

Domain: $-4 \leq x \leq 6$
or $[-4, 6]$



Answer: $(-\infty, 1) \cup (3, \infty)$

Trigonometric Functions (VERY IMPORTANT FOR ENGINEERS!) See Appendix D and the "Formula Page" (before the Title Page in your text).

Example: Solve for x : $\sin(2x) = \cos x$

$$\sin(2x) - \cos x = 0$$

$$2 \sin x \cos x - \cos x = 0$$

$$\cos x (2 \sin x - 1) = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2} + 2\pi n$$
$$x = \frac{3\pi}{2} + 2\pi n$$

$$2 \sin x - 1 = 0$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6} + 2\pi n$$
$$x = \frac{5\pi}{6} + 2\pi n$$

On Your Own: If $f(x) = \frac{1}{x-1}$ and $g(x) = \tan x$, find:

$$\begin{aligned} \text{a) } f(g(x)) &= f(\tan x) \\ &= \frac{1}{\tan x - 1} \end{aligned}$$

$$\frac{1}{\tan x - 1}$$

$$\begin{aligned} \text{b) } g(f(x)) &= g\left(\frac{1}{x-1}\right) \\ &= \tan\left(\frac{1}{x-1}\right) \end{aligned}$$

$$\tan\left(\frac{1}{x-1}\right)$$

$$\text{c) } \frac{f(x+h) - f(x)}{h} \text{ (and simplify)}$$

$$\frac{-h}{(x+h-1)(x-1)}$$

$$= \frac{\frac{1}{x+h-1} - \frac{1}{x-1}}{h}$$

$$= \frac{1}{h} \left(\frac{1 - 1}{(x-1)(x+h-1)} \right) = \frac{1}{h} \left(\frac{-h}{(x-1)(x+h-1)} \right) = \frac{-1}{(x+h-1)(x-1)}$$