1 2.5: Continuity

Definitions:

\( f \) is continuous at \( x = a \)

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Removable Discontinuities

**Source for understanding:** Maplet “Left and Right Hand Limits and Continuities”, located at http://calclab.math.tamu.edu/maple/maplets/ (NetID login)
Theorems:

Limits inside Continuous Functions

Continuity of Polynomial/Rational Functions

Intermediate Value Theorem

Examples:

If $f(x) = \begin{cases} 
1 - x & \text{if } x \geq 1 \\
-x & \text{if } x < 1 
\end{cases}$

determine whether $f$ is continuous at $x = 1$ or not. Explain your answer precisely. Is $f$ continuous from the left or right? Does $f$ have a removeable discontinuity?
\[
\lim_{x \to 1} \sqrt{\frac{x^2 + 2x - 3}{x - 1}} =
\]

Is there a real solution to the equation \( x^5 - x^2 + 2x = 6 \)? If so, find the value of \( a \) such there is a solution in the interval \([a, a + 1]\).

**On Beyond Average:** Let \( A \) be a constant, and consider the function

\[
f(x) = \begin{cases} 
4x + A & \text{if } x < 2 \\
2 & \text{if } x = 2 \\
x^2 - Ax + 1 & \text{if } x > 2
\end{cases}
\]

Determine the value of \( A \) for which \( \lim_{x \to 2} f(x) \) exists or explain why there is no such value \( A \).