1 3.5: Chain Rule

From 3.4: we know \( \frac{d}{dx}(\sin x) = \cos x \). Does \( \frac{d}{dx}(\sin 2x) = \cos 2x \)?

Recall: The composition of 2 functions \( f \) and \( g \) is defined by

Define \( f \) and \( g \) for the above function.

The Chain Rule: If \( f \) and \( g \) are differentiable functions, \( y = f(u) \) and \( u = g(x) \), then

\[
\frac{dy}{dx} =
\]

An alternate version of the Chain Rule states that \( \frac{d}{dx} f(g(x)) =
\]

Examples:

Find the derivatives of the following:
\( f(x) = \sin(x^2) - \sin^3 x \)
\( y = \sqrt{\cos^2(3x) + 1} \)
Differentiate the following:

\[ f(x) = x^2 \tan(3x) \quad \quad \quad \quad \quad \quad y = \frac{2x + 1}{\sin^2 x} \]

Given \( f(1) = 2, f(2) = 2\sqrt{2}, f'(1) = 1, f'(2) = \frac{1}{2}, g(1) = 2, g(2) = \frac{5}{2}, g'(1) = \frac{3}{4}, g'(2) = 0, \) and \( u(x) = f(g(x)), \) find \( u'(1) \)

**On Beyond Average:** Compute \( \lim_{h \to 0} \frac{\tan^2 \left( \frac{\pi}{4} + h \right) - 1}{h}. \)