1 5.2: Maxima and Minima

Definitions:

\( f \) has a \textbf{relative maximum} at \( x = a \) if and only if

\( f \) has a \textbf{relative minimum} at \( x = a \) if and only if

\textbf{Fermat’s Theorem}: If \( f \) has a relative maximum or relative minimum at \( x = a \) and \( f \) is differentiable at \( x = a \), then

\textbf{More Definitions}:

\( f \) has a \textbf{critical value} at \( x = a \) if and only if

\( f \) has an \textbf{absolute maximum} at \( x = a \) if and only if

\( f \) has an \textbf{absolute minimum} at \( x = a \) if and only if

\textbf{Extreme Value Theorem} If \( f \) is continuous on a closed, bounded interval, then

Graphical examples to show that each of the conditions must hold to guarantee the conclusion:
Examples:

Find the absolute maximum and absolute minimum of \( f(x) = 4x^3 - 15x^2 + 12x + 7 \) on the interval \( 0 \leq x \leq 3 \).

Find the absolute maximum and absolute minimum of \( f(x) = \frac{\ln x}{x^2} \) on the interval \((0, 3)\).

On Beyond Average: Suppose \( f \) is differentiable for all \( x \), and that \( f \) has exactly one critical value at \( x = 1 \). Find all the critical values of \( h(x) = f(x^2) \).