1 6.1-6.2: Area

Introduction:

The second historical problem of Calculus involves finding the area under the graph of a function. Suppose we want to find the area under the graph of \( f(x) = e^{-x^2} \) between \( x = 0 \) and \( x = 2 \) (denoted by \( \int_0^2 e^{-x^2} \, dx \)). We cannot find the exact area geometrically (or even using techniques you may have previously learned in Calculus!), so we approximate the area using rectangles (see figures in class):

What happens if more rectangles are used?

How many rectangles are needed to get the exact area?
Definitions:

partition:

$\Delta x_i$:

$||P||$:

$x_i^*$:

A Riemann Sum

The area under the graph of a positive function $f$ from $x = a$ to $x = b$:

Examples:

Given $f(x) = x^2 + x$, write and compute a Riemann Sum to approximate the area under $f$ from $x = 0$ to $x = 3$ using a partition $P = \{0, 1.5, 2.5, 3\}$. Let $x_i^*$ = the midpoint of each subinterval.
On Beyond Average: Find the exact area under the graph of $f$ from $x = 0$ to $x = 3$. (HINT: use $n$ equally spaced partitions and take $x_i^*$ =the right endpoint of each rectangle. Then let $n \to \infty$)