1 6.4-The Fundamental Theorem of Calculus

Fundamental Theorem of Calculus, part I: Given $f$ is a continuous function on $[a, b]$ and $g(x) = \int_{a}^{b} f(x) \, dx$, proof that $g$ is differentiable:

Fundamental Theorem of Calculus, part II:
Examples:

Find $F'(x)$ given:

$$F(x) = \int_\pi^x \frac{\sin t}{t} \, dt$$

$$F(x) = \int_x^3 \sqrt{1 + t^3} \, dt$$

Compute

$$\int_1^2 \left( x + \frac{1}{x^2} \right)^2 \, dx$$
Compute \( \int_0^\pi (\sin x + x^3 - e^x) \, dx \)

Find the exact area under the graph of \( f(x) = \frac{2}{1 + x^2} \) from \( x = 0 \) to \( x = \sqrt{3} \).

On Beyond Average: Compute \( \lim_{{x \to 0}} \frac{\int_0^x \sin^3 t \, dt}{x^4} \).