1 3.11: Linear and Quadratic Approximation

**Purpose:** To understand Linear (Differential) and Quadratic Approximation to a function near a certain point.

**Recall:** Given \( y = f(x) \), the tangent line at \( x = a \) is the best approximation to the graph of \( f \) “near” \( x = a \).

**Why?**

Formula for the tangent line:

\[
L(x) = f(a) + f'(a)(x - a)
\]

Therefore, if we want to approximate values of \( f \) near a given \( x \)-value \( (a) \), we can use the tangent line to obtain these approximations.

**Example:**

Use the linear approximation at \( x = \frac{27}{8} \) of an appropriate function to estimate \( \sqrt{3} \).

**A different view:** Because of the Linear Approximation, for values of \( x \) “near” \( x = a \), we have

\[
f(x) \approx L(x) = f(a) + f'(a)(x - a)
\]
Examples

The circumference around the middle of a sphere is measured to be 40cm, with a possible error of ±1 cm. Use differentials to estimate the possible error in the volume of the sphere.

Quadratic Approximation:

On Beyond Average: Given that the linear approximation to \( f \) at \( x = 2 \) is \( L(x) = \frac{3}{2} - \frac{5}{2}x \), what are \( f(2) \) and \( f'(2) \)?