Warm-Up

Given the graph of the DERIVATIVE of $f$ at left, on which interval is $f$ increasing?

(a) $[-2, 4]$ only
(b) $(-\infty, -2] \cup [4, \infty)$
(c) None of these
(d) $(-\infty, 1]$ only
(e) $(-\infty, -4] \cup [1, 7]$
1 5.1: Graphical Interpretation of $f$, $f'$, and $f''$

Graphical Interpretations of $f'$:
- If $f'(x) > 0$ for all $x \in (a, b)$ then $f$ is increasing for all $x \in (a, b)$
- If $f'(x) < 0$ for all $x \in (a, b)$ then $f$ is decreasing for all $x \in (a, b)$

Example: Draw a function $f$ from $(1,0)$ to $(4,5)$ with $f' > 0$:

![Graphs of function $f$ with different concavities]
Definitions:

a differentiable function $f$ is **concave up** on an interval $(a, b)$ if and only if $f'$ increasing on $(a, b)$

a differentiable function $f$ is **concave down** on an interval $(a, b)$ if and only if $f'$ decreasing on $(a, b)$

Therefore...

If $f''(x) > 0$ for all $x \in (a, b)$, then $f'$ inc and $f$ conc up for all $x \in (a, b)$

If $f''(x) < 0$ for all $x \in (a, b)$, then $f'$ dec and $f$ conc down for all $x \in (a, b)$

If $f''(x) = 0$ at $x = c$, then $f$ MAY change concavity at $x = c$ and, if so, $(c, f(c))$ is called an **inflection point**.
Example: PROPERTIES OF THE GRAPH Maplet

The numbers appearing in your answers must be chosen from the following list:
-10, 0.0, -0.05, 0.1, -1.92, -1.90, 1.03, 2.00, 4.42, 6.85, 9.00, 10.00

On what interval(s) is $f$ increasing?

On what interval(s) is $f$ concave up?

Which of these graphs is $f$? Click below the Plot.

Which of these graphs is $f''$? Click below the Plot.
On Beyond Average: Sketch the graph of a continuous function which satisfies the following:

- $f'(x) < 0$ for $x \in (-1, 1)$
- $f''(x) > 0$ for $x \in (-\infty, -1) \cup (1, \infty)$
- $f(-1) = 4$, $f(1) = 0$
- $f''(x) < 0$ for all $x \neq 1$