6.3: The Definite Integral

Note that, although we assumed \( f \) was positive to illustrate the approximating rectangles, the definition can still be calculated even if \( f \) is not always positive.

**The Definite Integral** of \( f \) from \( x = a \) to \( x = b \) is given by

If \( f > 0 \), \( \int_a^b f(x) \, dx \) gives us the area under the graph of \( f \) from \( x = a \) to \( x = b \).

**Equally Spaced Partitions:** Let \( n \) be the number of equally-spaced subintervals of \([a, b]\).

Then \( \Delta x_i = \)

and \( \int_a^b f(x) \, dx = \)

**Properties of Definite Integrals** (pp 383-385)

(NOTE: Some of the more useful properties for future sections are #2, 3, 5, and 8)
Given \( f(x) = x^2 - 3x + 1 \), find the exact value of \( \int_1^3 f(x) \, dx \) from the definition.
Rewrite $\int_{-2}^{5} f(x) \, dx - \int_{3}^{5} f(x) \, dx + \int_{3}^{7} f(x) \, dx$ as a single integral.

**On Beyond Average:** Compute $\int_{0}^{4} \left( |x - 3| + \sqrt{16 - x^2} \right) \, dx$

(HINT: Use properties to split up, then remember that integral = area since both functions are positive on $[0, 4]$)