11.1: Three-Dimensional Coordinates

Recall: The $x$-$y$ plane is the set of all points $(x_0, y_0)$, where $x_0$ refers to the horizontal and $y_0$ refers to the vertical.

Now: 3-dimensional space is set of points $(x, y, z)$. 

The distance between 2 points in 3-dimensional space is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Proof:

$$d = \sqrt{c^2 + (z_2 - z_1)^2}$$

where

$$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
The set of points equidistant from a given point is called a \textit{sphere}.

Therefore, we can derive the equation as follows:

Equation for all \((x, y, z)\) whose distance from \((x_0, y_0, z_0)\) is a constant \(r\).

\[
(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2
\]

\((x_0, y_0, z_0) = \text{center} \quad r = \text{radius}\)

Equation of a \textit{sphere}
Examples:

Describe the set of points which satisfy the equation \( x^2 + y^2 + z^2 - 4x + 6y - 3z - 5 = 0 \). Sketch the graph below.

Sphere: Complete the Square!

\[
\begin{align*}
  x^2 - 4x + 4 + y^2 + 6y + 9 + z^2 - 3z + \frac{9}{4} &= 5 + 4 + 9 + \frac{9}{4} \\
  (x-2)^2 + (y+3)^2 + (z-\frac{3}{2})^2 &= \frac{81}{4} = r^2
\end{align*}
\]

Center is \((2, -3, \frac{3}{2})\)

Radius is \(\frac{9}{2}\)
Describe the set of points in 3-dimensional space which satisfy the equation \( y = mx + b \), where \( m \) and \( b \) are constants. Sketch the graph below.
Describe the set of points in 3-dimensional space which satisfy the equation $z^2 + z^2 = 4$. 

\textit{cylinder}
Describe the set of all points which are equidistant from the two points (1, 2, 3) and (3, 5, 7). Find the equation of this object.

We want all \((x, y, z)\) such that distance from \((x, y, z)\) to \((1, 2, 3)\) is equal to the distance from \((x, y, z)\) to \((3, 5, 7)\).

(We know \((2, 3\frac{1}{2}, 5)\) satisfies this — midpoint of the line segment.)

i.e. \[\sqrt{(x-1)^2 + (y-2)^2 + (z-3)^2} = \sqrt{(x-3)^2 + (y-5)^2 + (z-7)^2}\]

\[x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 10y + 25 + z^2 - 14z + 49\]

\[4x + 6y + 8z = 69\] plane

2-D: 

3-D: 

...