11.3: Torque and Cross-Product

Definitions: The torque of a vector $\mathbf{F}$ about a vector $\mathbf{r}$

The cross-product of vectors $\mathbf{a}$ and $\mathbf{b}$:

Note that $\mathbf{a} \times \mathbf{b}$ is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$: If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, and $\mathbf{a} \times \mathbf{b} = \langle x_1, x_2, x_3 \rangle$, then we have:

The cross-product of vectors $\mathbf{a}$ and $\mathbf{b}$ (ONLY in $\mathbb{R}^3$) is given by $\mathbf{a} \times \mathbf{b} =$
NOTES:

1. Simple calculation method:

2. Geometric significance:

3. $|\mathbf{a} \times \mathbf{b}| =$

4. Useful Properties (all listed on p668):

Examples:

Find $\mathbf{a} \times \mathbf{b}$ if $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} - \mathbf{j} + 7\mathbf{k}$
Given the points $P(1,0,−1)$, $Q(2,4,5)$, and $R(3,1,7)$ find a vector orthogonal to the plane containing these points.

Find the area of $\triangle PQR$. 
Find the volume of the parallelepiped whose corner is formed by the vectors \( \mathbf{a} = \langle 2, 3, -2 \rangle \), \( \mathbf{b} = \langle 1, -1, 0 \rangle \), and \( \mathbf{c} = \langle 2, 0, 3 \rangle \).

A wrench 0.5m long is applied to a nut with a force of 80N (See picture below). Because of limited space, the force must be exerted straight upward. How much torque is applied to the nut?