1 7.2: Volume by Slicing

The volume of a prism with a base of area \( B \) and a height \( h \) is given by

If the height is not constant, we can, in many cases, “slice” the solid thin enough that the height is constant.

**Examples:** Derive (prove) the formula for the volume of a hemisphere with radius \( r \).

Find the volume of the solid formed by rotating the region bounded by the curves \( y = \sqrt{x - 1}, \ y = 0, \) and \( x = 3 \) about the \( x \)-axis.
Find the volume of the solid formed by rotating the region on the previous page about the $y$-axis.

Find the volume of the solid formed by rotating the region bounded by the curves $y = \cos x$, $y = \sin x$, $x = 0$, and $x = \frac{\pi}{4}$ about the $x$-axis.
Set up, but do not evaluate, an integral to find the volume of the solid formed by rotating the region bounded by \( y = \ln x \), \( y = 0 \), and \( x = e^2 \) about the line \( x = -1 \).

The base of a solid is the region bounded by \( y = x^2 \) and \( y = 2 - x^2 \). Cross-sections perpendicular to the \( x \)-axis are squares. Find the volume of the solid.