1 Section 10.4

Key Concepts:

1. **Alternating Series Test**: If $a_n$ decreases and $a_n \rightarrow 0$, then $\sum (-1)^n a_n$ is convergent.

2. **Alternating Series Estimation**: If $\sum (-1)^n a_n$ converges to $s$ and $s = \sum_{n=1}^{N} (-1)^n a_n$ ($a_n > 0$), then $|s - s_N| \leq a_{N+1}$.

3. **Absolute Convergence**: If $\sum |a_n|$ is convergent, then $\sum a_n$ is absolutely convergent (and hence, convergent).

4. **Ratio Test**: Let $L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$. If $L > 1$, $\sum a_n$ is divergent; if $L < 1$, $\sum a_n$ is absolutely convergent.

Examples:

1. Determine if the following series are absolutely convergent, convergent (but not absolutely convergent), or divergent. State the test used and show all conditions are satisfied.

   (a) $\sum_{n=1}^{\infty} \frac{(-1)^n + 1}{2n + 1}$
   (b) $\sum_{n=1}^{\infty} \frac{2^{2n}}{3^{n+1}}$
   (c) $\sum_{n=0}^{\infty} \frac{(-3)^n}{n!}$

2. The hundredth partial sum of 1c above is $s_{100} \approx 0.049787$. Estimate how close this is to the actual sum of the series.

3. How many terms of the series in 1a above are needed to approximate the exact sum to within 0.001?

2 Section 10.5

Key Concepts:

1. **Power Series**: A series of the form $\sum c_n (x - a)^n$

2. **Interval of Convergence**: The set of all values of $x$ for which $\sum c_n (x - a)^n$ is convergent. Possibilities:
   - (a) only when $x = a$
   - (b) $x \in (-\infty, \infty)$
   - (c) $x \in (a - r, a + r)$ for some positive $r$ (possibly including endpoints)

3. **Radius of Convergence**: The largest value of $r$ such that, if $x \in (a - r, a + r)$, then $\sum c_n (x - a)^n$ is convergent. (Case (a): $r = 0$, Case (b): $r = \infty$)

Examples:

1. Find the radius and interval of convergence of each of the following:

   (a) $\sum_{n=0}^{\infty} \frac{(x + 2)^n}{2^n}$
   (b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n + 1}$
   (c) $\sum_{n=2}^{\infty} \frac{3^n (x - 1)^n}{n \ln n}$
   (d) $\sum_{n=0}^{\infty} \frac{2^n (x - 3)^n}{n!}$

2. Given $\sum c_n (x - 2)^n$ is convergent when $x = 5$ and divergent when $x = -4$, determine if the following are convergent, if possible:
Section 10.6

Key Concepts:

1. If \( \sum c_n(x-a)^n \) converges to \( f(x) \) with radius of convergence \( R \), then \( \sum c_n n(x-a)^{n-1} \) converges to \( f'(x) \) with radius of convergence \( R \).

2. If \( \sum c_n(x-a)^n \) converges to \( f(x) \) with radius of convergence \( R \), then \( \sum \frac{c_n}{n+1} (x-a)^{n+1} \) converges to \( \int f(x) \, dx \) with radius of convergence \( R \).

3. KEY to creating many power series: Geometric series formula \( \sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \).

Examples:

1. Find a power series for the following functions and determine the radius and interval of convergence:
   (a) \( f(x) = \arctan x \)
   (b) \( f(x) = \frac{1}{(2-x)^2} \)
   (c) \( f(x) = \ln(1+x^2) \)

2. (a) Use a power series to compute
   \[ \int_{0}^{1} \frac{1}{1+x^4} \, dx. \]
   (b) How many terms of the series are needed to approximate the integral to within 0.01?