1 Section 7.3

Key Points:

1. \[ V_{\text{shell}} = 2\pi rh\Delta r, \quad r = |\text{var} - \text{axis}|, \quad h = T - B \text{ or } R - L \]

2. Which method? Compare variable of integration to axis of rotation:
   (a) SAME = slices (7.2)
   (b) DIFFERENT = shells (7.3)

Examples:

1. Find the volume of the solid formed by rotating the regions bounded by the following curves about the given axis:
   (a) \[ y = 9 - \frac{1}{2}x^2 \text{ and } y = (x - 3)^2 \text{ about the } y\text{-axis}. \]
   (b) \[ x = \frac{\sqrt{y}}{2}, \quad y = 4, \quad x = 0 \text{ about the } y\text{-axis}. \]
   (c) \[ y = x^2, \quad x = y^2 \text{ about the line } y = 1. \]
   (d) \[ y = e^x, \quad y = e^{2x}, \quad x = 1 \text{ about the line } x = 1. \]
   (e) \[ y = e^x, \quad y = e^{2x}, \quad x = 1 \text{ about the line } y = 1. \]
   (f) \[ y = \frac{1}{1 + x^2}, \quad x = \sqrt{2y}, \quad x = 0 \text{ about the } y\text{-axis}. \]
   (g) \[ x^2 + y^2 = 10, \quad x + y = 4 \text{ about the } x\text{-axis}. \]

2 Section 7.4

Key Points:

1. Force = \( f(x) \) (\( x \) displacement), then \( W = \int_a^b f(x) \, dx \).

2. "Water Pumping": partition along vertical (called \( y \) here). \( W = \int_a^b \text{density} \cdot \text{volume} \cdot \text{distance} \)
   (a) \( y = 0 \) can be top, bottom, or center (circular ends)
   (b) \( \text{volume} = \text{length} \cdot x(y) \cdot dy \) where \( x(y) = \text{width} \) found geometrically or with coordinate system

Examples:

1. The natural length of a spring is 12 inches. If a force of 20 lbs is required to hold the spring at a length of 15 inches, find the work done in stretching the spring from its natural length to a length of 18 inches.

2. The natural length of a spring is 20 cm. The work done in stretching the spring to a length of 30 cm is 10 J. Find the work done in stretching the spring from a length of 25 cm to a length of 35 cm.

3. The tanks below are full of water. Find the work required to pump the water out of the tank. (Use \( \rho g \) for weight density).
   (a) 30 feet long, cross-section semicircle with radius 12 feet.
(b) 10 m long, triangular cross-section with base 8 m and height 3 m.

(c) hemisphere with radius 4m with a 1m pipe extending up from the top.

4. 60 meters of cable hang from a pulley. If the cable density is 4 kg per meter, find the work required to pull 40 meters of the cable to the top.