1 Section 7.5

Key Points:

1. \( f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) \, dx \)

2. If \( f > 0 \), \( f_{\text{avg}} \) is the height of the rectangle whose area equals the area under the graph of \( f \) from \( x = a \) to \( x = b \). (NOTE: if \( f \) is continuous, the rectangle will always intersect the graph of \( f \)).

Examples:

1. Find the average value of the given function over the given interval:
   
   (a) \( f(x) = \frac{2}{x\sqrt{x}} \) over \([1, 4]\)
   
   (b) \( f(x) = x\sqrt{x^2 + a^2} \) over \([0, a]\)
   
   (c) \( f(x) = \frac{1}{4-3x} \) over \([-2, 1]\)
   
   (d) \( f(x) = \frac{e^{2x} - 1}{e^x} \) over \([0, 3]\)
   
   (e) \( f(x) = \sin^3 x \cos x \) over \([0, \frac{\pi}{2}]\)
   
   (f) \( f(x) = \frac{1}{1 + x^2} \) over \([-1, 1]\)

2 Section 8.1

Key Points:

1. \( \int u \, dv = uv - \int v \, du \)

Examples:

1. Compute the following integrals:
   
   (a) \( \int x^2 \sin(3x) \, dx \)
   
   (b) \( \int y^3 e^{-y^2} \, dy \)
   
   (c) \( \int_0^{1/2} \sin^{-1}(2x) \, dx \)
   
   (d) \( \int t^3 \sqrt{1-t^2} \, dt \)
   
   (e) \( \int_1^e \frac{\ln x}{x^2} \, dx \)
   
   (f) \( \int e^{3x} \cos(2x) \, dx \)

2. Find the volume obtained by rotating the region bounded by \( y = \sin x, y = \cos x, x = 0 \), and \( x = \frac{\pi}{4} \) about the y-axis.
3 Section 8.2

Key Points:

1. Know the integrals of all six trig functions
2. Know identities to integrate the squares of all six trig functions
3. Other trig integrals: when possible, “save” a $du$ and rewrite the rest of the integral in terms of $u$ using identities
4. Product to Sum integrals: Based on the Angle Addition identities

Examples:

1. Integrate the following:

(a) $\int \cos^3(2x) \, dx$

(b) $\int_0^{\pi/2} \sin^3 x \sqrt{\cos x} \, dx$

(c) $\int \sec^3 x \tan^3 x \, dx$

(d) $\int_{\pi/6}^{\pi/2} \csc^{3/2} x \cot^3 x \, dx$

(e) $\int \sin(6x) \sin(4x) \, dx$

(f) $\int \sin(2x) \cos(4x) \, dx$

2. Find the volume of the solid obtained by rotating the region under the graph of $y = \sin^2 x$ and above the $x$ axis (one period only) about the $x$-axis.