1 Section 8.3

Key Points:

1. integral with $a^2 - x^2$: use $x = a \sin \theta$
2. integral with $x^2 + a^2$: use $x = a \tan \theta$
3. integral with $x^2 - a^2$: use $x = a \sec \theta$
4. Return to $x$: use reference triangles to simplify expressions such as $\tan \left( \sin^{-1} \left( \frac{x}{a} \right) \right)$

Examples:

1. Compute the following integrals:

   (a) $\int \frac{\sqrt{9 - x^2}}{x} \, dx$
   (b) $\int \frac{\sqrt{5}}{\sqrt{x^2 + 5}} \, dx$
   (c) $\int \frac{\sqrt{3x^2 - 9}}{x} \, dx$
   (d) $\int_0^2 \frac{dx}{(4x^2 + 9)^{3/2}}$
   (e) $\int_2^3 \frac{x \, dx}{\sqrt{9x^2 - 16}}$
   (f) $\int \sqrt{3 - 2x - x^2} \, dx$

2 Section 8.4

Key Points:

1. Only works on rational functions!!! Factor Denominator and “Break Down” the fraction using individual denominators, set up numerators as follows:

   (a) Distinct Linear Factors: constants in numerators $\frac{A}{x - a} + \cdots$
   (b) Repeated Linear Factors: increase powers with constants in numerators $\frac{A}{(x - a)} + \frac{B}{(x - a)^2} + \cdots$
   (c) Prime Quadratic Factors: linear terms in numerators $\frac{Ax + B}{ax^2 + bx + c} + \cdots$
   (d) Repeated Prime Quadratic Factors: increase powers with linear terms in numerators $\frac{Ax + B}{ax^2 + bx + c} + \frac{Cx + D}{ax^2 + bx + c} + \cdots$
   (e) Improper rational function (degree numerator > degree denominator): Divide, then apply above to remainder

2. Solve for numerators (matching powers and/or using “nice” values of $x$) and integrate

Examples:

1. Write the partial fraction decomposition of the proper rational function $\frac{p(x)}{(x - 3)^3(x^2 + 2x + 5)(x + 5)}$ (do not try to solve)

2. Compute the following integrals:

   (a) $\int_1^3 \frac{dx}{x^2 + 3x}$
   (b) $\int \frac{x^2 - 7x + 9}{(x - 1)^2(x + 2)} \, dx$
   (c) $\int \frac{x^5 - x^4 - 3x^3 - 2x^2 + 2x + 5}{x^4 + x^3} \, dx$
   (d) $\int \frac{x^2}{x^4 - 81} \, dx$
   (e) $\int \frac{x^3 + 8}{x^3 - 8} \, dx$