1 Section 10.2-10.4

Key Concepts: SUMMARY OF CONVERGENCE TESTS

1. **Test for Divergence** (show \(a_n \not\to 0\); then series is divergent)

2. **Ratio Test** (let \(L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|\). series is divergent if \(L > 1\) and absolutely convergent if \(L < 1\))

3. **Alternating Series Test** (show \(|a_n| \to 0\); then alternating series is convergent. Test \(\sum |a_n|\) to determine absolute convergence)

4. **Limit Comparison Test** (let \(L = \lim_{n \to \infty} \left| \frac{a_n}{b_n} \right|\), then both series are convergent or both divergent if \(0 < L < \infty\))

5. **P-Test** (\(\sum \frac{1}{n^p}\) is convergent if and only if \(p > 1\). Usually used to create \(b_n\) in #4 or #6)

6. **Comparison Test** (if \(a_n < b_n\) and \(\sum b_n\) is convergent, then \(\sum a_n\) is convergent. If \(b_n < a_n\) and \(\sum b_n\) divergent, then \(\sum a_n\) divergent)

7. **Integral Test** (if \(a_n = f(n)\), then \(\sum a_n\) is convergent if and only if \(\int_# f(x) \, dx\) converges)

Examples:

1. Determine if the following series are absolutely convergent, convergent (but not absolutely) or divergent. State the name of the test used and show all conditions of the test are met.

   (a) \(\sum_{n=1}^{\infty} \frac{n^2 + 1}{3^n}\)
   (b) \(\sum_{n=1}^{\infty} \frac{2^n - 1}{2^n}\)
   (c) \(\sum_{n=1}^{\infty} \frac{n^3}{n^4 + 1}\)
   (d) \(\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n}{2n^2 + 3}\)
   (e) \(\sum_{n=1}^{\infty} \frac{1}{2n - n}\)
   (f) \(\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n} + 1}\)
   (g) \(\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\tan^{-1} n}{n}\)
   (h) \(\sum_{n=1}^{\infty} \frac{\sin n}{n^2}\)
   (i) \(\sum_{n=1}^{\infty} \frac{2^{n^2}}{n!}\)
   (j) \(\sum_{n=1}^{\infty} \frac{3 \cdot 6 \cdot 9 \cdots 3n}{(n + 1)!}\)
   (k) \(\sum_{n=1}^{\infty} \frac{5 + (-1)^{n+1}}{3^n}\)