1 11.3: Torque and Cross-Product

**Definitions:** The **torque** of a vector \( \mathbf{F} \) about a vector \( \mathbf{r} \)

The **cross-product** of vectors \( \mathbf{a} \) and \( \mathbf{b} \):

Note that \( \mathbf{a} \times \mathbf{b} \) is orthogonal to both \( \mathbf{a} \) and \( \mathbf{b} \): If \( \mathbf{a} = \langle a_1, a_2, a_3 \rangle \), \( \mathbf{b} = \langle b_1, b_2, b_3 \rangle \), and \( \mathbf{a} \times \mathbf{b} = \langle x_1, x_2, x_3 \rangle \), then we have:

The **cross-product** of vectors \( \mathbf{a} \) and \( \mathbf{b} \) (ONLY in \( \mathbb{R}^3 \)) is given by \( \mathbf{a} \times \mathbf{b} = \)
NOTES:

1. Simple calculation method:

2. Geometric significance:

3. $|a \times b| =$

4. Useful Properties (all listed on p668):

Examples:

Find $a \times b$ if $a = i + 2j - k$ and $b = 3i - j + 7k$
Given the points $P(1, 0, -1)$, $Q(2, 4, 5)$, and $R(3, 1, 7)$ find a vector orthogonal to the plane containing these points.

Find the area of $\triangle PQR$. 
Find the volume of the parallelipiped whose corner is formed by the vectors \( \mathbf{a} = (2, 3, -2) \), \( \mathbf{b} = (1, -1, 0) \), and \( \mathbf{c} = (2, 0, 3) \).

**On Beyond Average:**

If \( \mathbf{a}, \mathbf{b}, \) and \( \mathbf{c} \) are 3-dimensional vectors, determine if each of the following is a vector, a scalar, or meaningless:

\[
(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} \quad (\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c} \quad (\mathbf{a} \cdot \mathbf{b}) \mathbf{c} \quad (\mathbf{a} \times \mathbf{b}) \times \mathbf{c} \quad \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})
\]

The vectors \( \mathbf{a}, \mathbf{b}, \) and \( \mathbf{c} = \mathbf{b} - \mathbf{a} \) all lie in the same plane as shown in the diagram below:

Circle all TRUE statements below:

\[
\mathbf{a} \times \mathbf{b} = 0 \quad \mathbf{a} \times \mathbf{b} \text{ points into the page}
\]

\[
(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = 0 \quad \mathbf{b} \times (\mathbf{a} \times \mathbf{c}) \text{ points in the direction of } -\mathbf{a}
\]