1 10.3: The Integral and Comparison Tests

(Series Convergence Tests continued)

II. The Integral Test:

Illustrated "Proof":

Find the values of $p$ for which $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent

(III. The P-Test)

Note that if $\sum_{n=1}^{\infty} a_n$ is convergent (to a sum $s$) by the Integral Test, then:
IV. The Comparison Test:

V. The Limit Comparison Test:

\[ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(n^{3/2} + 1)} \]
On Beyond Average:

Suppose \( \sum_{n=1}^{\infty} a_n \) and \( \sum_{n=1}^{\infty} b_n \) are series with positive terms and \( \lim_{n \to \infty} \frac{a_n}{b_n} = 0 \). Circle the true statement(s):

- If \( \sum_{n=1}^{\infty} b_n \) is convergent, then \( \sum_{n=1}^{\infty} a_n \) is convergent.
- If \( \sum_{n=1}^{\infty} b_n \) is divergent, then \( \sum_{n=1}^{\infty} a_n \) is divergent.
- If \( \sum_{n=1}^{\infty} a_n \) is convergent, then \( \sum_{n=1}^{\infty} b_n \) is divergent.
- If \( \sum_{n=1}^{\infty} b_n \) is convergent, then \( \sum_{n=1}^{\infty} a_n \) is divergent.
- There is not enough information.

Explain why the series \( \sum_{n=1}^{\infty} \frac{\ln n}{n^3} \) is convergent. Estimate the maximum possible error when using \( s_{10} \) as an approximation for the sum of the series.