11.1: Three-Dimensional Coordinates

**Recall:** The $x$-$y$ plane is the set of all points $(x_0, y_0)$, where $x_0$ refers to the horizontal and $y_0$ refers to the vertical.

Now: 3-dimensional space is the set of all points $(x_0, y_0, z_0)$.

The distance between two points in 3-dimensional space is given by

$$d = \sqrt{(ax)^2 + (by)^2 + (cz)^2}$$

**Proof:**

$$d = \sqrt{(ax)^2 + (by)^2 + (cz)^2}$$

$$= \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$$
The set of points equidistant from a given point is called a sphere.

Therefore, we can derive the equation as follows:

Given center \((x_0, y_0, z_0)\) and radius \(r\),

Let \((x, y, z)\) be a point on the sphere. Then the distance from \((x, y, z)\) to \((x_0, y_0, z_0)\) is \(r\).

\[
\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} = r
\]

or

\[
(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2
\]
Examples:

Describe the set of points which satisfy the equation \( x^2 + y^2 + z^2 - 4x + 6y - 3z - 5 = 0 \). Sketch the graph below.

\[
\begin{align*}
\quad & x^2 - 4x + 4 + y^2 + 6y + 9 + z^2 - 3z + \frac{9}{4} = 5 + 4 + 9 + \frac{9}{4} \\
(\mathbf{x - 2})^2 + (\mathbf{y + 3})^2 + (\mathbf{z - \frac{3}{2}})^2 &= 18 + \frac{9}{4} \\
\frac{24}{4} + \frac{9}{4} &= \frac{61}{4} \quad \text{radius}
\end{align*}
\]

Sphere with center \((2, -3, \frac{3}{2})\)

radius \(= \frac{9}{2}\)
Describe the set of points in 3-dimensional space which satisfy the equation $y = mx + b$, where $m$ and $b$ are constants. Sketch the graph below.

A plane parallel to $z$-axis (⊥ xy plane)
Describe the set of all points which are equidistant from the TWO points (1, 2, 3) and (3, 5, 7). Find the equation of this object.

Let \((x, y, z)\) be a point on this surface.

\[
\sqrt{(x-1)^2 + (y-2)^2 + (z-3)^2} = \sqrt{(x-3)^2 + (y-5)^2 + (z-7)^2}
\]

\[
x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 10y + 25 + z^2 - 14z + 49 - 1 - 4 - 9
\]

\[
x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 10y + 25 + z^2 - 14z + 49 - 1 - 4 - 9
\]

\[
4x + 6y + 8z = 69
\]

A plane \( \perp \) to and bisecting the line segment from \((1, 2, 3)\) to \((3, 5, 7)\).
On Beyond Average:

Find an equation of the plane parallel to the $x$-axis which contains the points $(0, 2, 0)$ and $(0, 0, 2)$.

\[ y + z = 2 \]

$x$ can be any value.
Determine if the triangle with vertices $P(4,2,1)$, $Q(3,3,1)$, and $R(3,2,3)$ is scalene, isosceles, or equilateral. Is $\triangle PQR$ a right triangle?

\[
\begin{align*}
distances \\
PQ &= \sqrt{(3-4)^2 + (3-2)^2 + (1-1)^2} = \sqrt{2} \\
QR &= \sqrt{(3-3)^2 + (2-3)^2 + (3-1)^2} = \sqrt{5} \\
PR &= \sqrt{(3-4)^2 + (2-2)^2 + (3-1)^2} = \sqrt{5}
\end{align*}
\]

$QR \equiv PR$  \text{isosceles}

right triangle? if $a^2 + b^2 = c^2$, then $\triangle$

\[
(\sqrt{2})^2 + (\sqrt{5})^2 \overset{?}{=} (\sqrt{5})^2
\]

$7 \neq 5 \times$

Not a right triangle
Describe the set of points in 3-dimensional space which satisfy the equation $x^2 + z^2 = 4$. (No y variable)

in x-z plane, circle radius 2

extend along $y$-axis

cylinder radius 2 along $y$-axis