Prep Work for The Fundamental Theorem of Calculus

Just like we defined a derivative function, we can define an integral function, provided we specify a starting point. Given a function $f(x)$, define $g(x) = \int_0^x f(t) \, dt$. This is illustrated in the following animation found at the University of Wisconsin-Eau Claire Math Department website: http://tinyurl.com/intfcn-gif.

(NOTE that we are changing the name of the $x$-axis to the $t$-axis and $x$ refers to a specific number on that axis.)

1. If $f(x) > 0$, what does the function $g$ give us?

2. Is $g$ still defined even if $f$ is not always $> 0$?

The function $g$ can be graphed on the TI calculator using the following steps:

1. Enter the function $f$ in $Y_1$.
2. For $Y_2$, press MATH, 9 to select fnInt(. Then press VARS, Y-VARS, Function..., $Y_1$, then press $X, 0, X$)
3. Press WINDOW to select an X-range, then ZOOM/ZoomFit to select a Y-range (NOTE: It will take some time to graph, so be patient!)

Use the technique above to graph the following pairs of functions. What do you notice?

1. $g(x) = \int_0^x 2x \, dx$; $F(x) = x^2$ (HINT: put $Y_1 = 2X$, $Y_2 = fnInt(Y_1, X, 0, X)$, and $Y_3 = X^2$)
2. $g(x) = \int_0^x 3x^2 \, dx$; $F(x) = x^3$
3. $g(x) = \int_0^x \sqrt{x} \, dx$; $F(x) = \frac{2}{3} x^{3/2}$

The “starting point” of your integral function does not have to be 0. Graph the following functions; how are they alike, and how are they different?

1. $g(x) = \int_2^x 3x^2 \, dx$; $F(x) = x^3$