

MATH 151
FALL 2006

SAMPLE EXAM III

Part I - Multiple Choice

1. Given $f(x) = x^3 \ln x$, find $f'(e)$

- a) e b) $3 + 3e^2$ c) e^2
d) $3e$ e) $4e^2$

2. The population of a bacteria colony triples every 5 hours. If the population follows an exponential growth model, find k .

- a) $\frac{\ln 2}{5}$ b) not enough information
c) $\frac{\ln 3}{5}$ d) 3 e) $\ln 5$

3. $\lim_{x \rightarrow -\infty} \sin^{-1} \left(\frac{e^x + \sqrt{3}e^{-x}}{2e^x + 2e^{-x}} \right) =$

- a) 30 b) 60 c) $\frac{\pi}{6}$
d) $\frac{\pi}{3}$ e) more than one answer is correct

4. $\frac{d}{dx}(\tan^{-1}(x^2)) =$

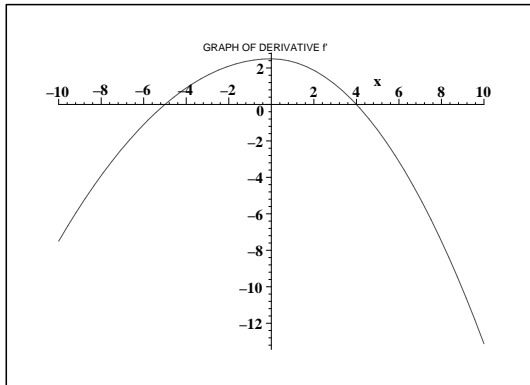
- a) $\frac{2x}{1+x^4}$ b) $\frac{2}{1+x^2}$ c) $-2x \csc(x^2) \cot(x^2)$
d) $2x \tan^{-1}(x^2) \sec^{-1}(x^2)$ e) $\frac{2x}{1+x^2}$

5. $\lim_{x \rightarrow 0} \frac{\tan(4x)}{\tan(7x)} =$

- a) $\frac{7}{4}$ b) $-\frac{7}{4}$ c) $-\frac{4}{7}$
 d) 1 e) $\frac{4}{7}$

6. The graph of the *DERIVATIVE* of a function is shown below. On which intervals is the original function f concave down?

CIRCLE ALL CORRECT CHOICES-THERE MAY BE MORE THAN ONE!



- a) $(-\infty, -5)$ b) $(-5, 0)$
 c) $(0, 4)$ d) $(4, \infty)$
 e) none of these intervals

7. Circle ALL the critical values of $f(x) = x(x - 1)^{\frac{1}{3}}$

NOTE: YOU MAY CIRCLE MORE THAN ONE CHOICE!

- a) 0 b) $-\frac{1}{3}$ c) $\frac{1}{4}$
 d) $\frac{3}{4}$ e) 1

8. Find the absolute maximum of $f(x) = \sin x + \cos x$ on the interval $\left[0, \frac{\pi}{3}\right]$.

(NOTE: $\sqrt{2} \approx 1.414$ and $\sqrt{3} \approx 1.73$)

- a) 1 b) 2 c) $\sqrt{2}$
 d) $\frac{\sqrt{3} + 1}{2}$ e) $\frac{\pi}{4}$

9. The inflection points of $f(x) = x^5 + 10x^4$ occur at which of the following?

- a) $x = 6$ only b) $x = -6$ only
 c) $x = 0, x = -8$ d) $x = 0, x = -6$
 e) $x = 0, x = 6$

10. To find the rectangle of perimeter 100 cm with the largest area, you would maximize which function?

a) $f(x) = x(50 - x)$

b) $f(x) = 2x + 2(100 - x)$

c) $f(x) = \frac{100}{x^2}$

d) $f(x) = 50 - 2x$

e) $f(x) = 2x + \frac{200}{x}$

11. Which is an antiderivative of $f(x) = 2\sqrt{x} + \frac{1}{x^2}$?

a) $\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{x} + C$

b) $\frac{4}{3}x^{\frac{3}{2}} - \ln(x^2) + C$

c) $\frac{1}{\sqrt{x}} - \frac{2}{x^3} + C$

d) $\frac{1}{\sqrt{x}} + \ln(x^2) + C$

e) $\frac{1}{\sqrt{x}} - \frac{1}{x} + C$

12. Write $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \frac{1}{64}$ in summation notation.

a.) $\sum_{n=1}^6 \frac{(-1)^{n+1}}{2^n}$

b.) $\sum_{n=1}^6 \frac{(-1)^n}{2^n}$

c.) $\sum_{n=1}^6 \frac{(-1)^{n+1}}{2n}$

d.) $\sum_{n=1}^6 \frac{(-1)^n}{2n}$

e.) $\sum_{n=1}^6 \frac{-1}{2^n}$

13. Find $\int_0^5 |x - 2| dx$

a.) $\frac{9}{2}$

b.) $\frac{7}{2}$

c.) $\frac{15}{2}$

d.) $\frac{11}{2}$

e.) $\frac{13}{2}$

14. Use the midpoint rule with $n = 4$ to approximate $\int_1^3 \ln x \, dx$

a.) $\ln \frac{3465}{256}$

b.) $\frac{1}{2} \ln \frac{3465}{256}$

c.) $\frac{1}{2} \ln \frac{32}{16}$

d.) $\ln \frac{32}{16}$

e.) None of the above.

15. Find $\lim_{x \rightarrow 0} \frac{\arctan x - x}{x^3}$

a.) $-\frac{1}{3}$

b.) $-\frac{1}{2}$

c.) 0

d.) ∞

e.) -6

16. Suppose it is given that $\int_1^3 f(x) \, dx = 7$, $\int_0^2 f(x) \, dx = 3$, $\int_2^3 f(x) \, dx = 13$, Find $\int_0^1 f(x) \, dx$.

a.) 9

b.) 3

c.) -2

d.) 7

e.) -7

Part II - Work Out Problems

17. Find the derivative of $f(x) = x^{\sec x}$

18. Find $\lim_{x \rightarrow 0} (1 - x)^{\frac{5}{x}}$

19. Sketch a graph with the following properties:

$$f'(x) > 0 \text{ when } x < 1, \quad f'(x) < 0 \text{ when } x > 1$$

$$f''(x) > 0 \text{ when } x < -2 \text{ and } x > 2, \quad f''(x) < 0 \text{ when } -2 < x < 2$$

$$\lim_{x \rightarrow -\infty} f(x) = -2, \quad \lim_{x \rightarrow \infty} f(x) = 0$$

20. Determine when the function $f(x) = xe^{4x}$ is increasing, decreasing, concave down, and concave up.

21. Find the dimensions of the largest rectangle that can be inscribed in a circle of radius 2.

22. The acceleration of a particle is given by $\mathbf{a}(t) = (1 + e^t)\mathbf{i} + (\cos t)\mathbf{j}$. If the initial velocity is \mathbf{i} and the initial position is \mathbf{j} , find the position of the particle at any time t .

23. A certain object cools at a rate (in $^{\circ}\text{F}/\text{min}$) of $1/2$ of the difference between its temperature and that of the surrounding air. If a room is kept at 75°F and the initial temperature of this object is 98° , find an expression of the temperature of the object t minutes later.

24. By introducing a deadly chemical, the population of a particular bacteria culture is decreasing over time. Suppose it has been determined that the rate of change of the population at time t minutes is $\frac{1}{3}$ of the population. If the initial size of the population is 1000 bacteria, how many bacteria are present after 3 minutes?

25. Find the following:

a.) $\arccos\left(\frac{1}{2}\right) = \underline{\hspace{2cm}}$

b.) $\sin\left(\arccos\left(-\frac{4}{5}\right)\right) = \underline{\hspace{2cm}}$

c.) $\arcsin\left(\sin\left(\frac{5\pi}{4}\right)\right) = \underline{\hspace{2cm}}$

d.) The domain of $\arccos(4x - 5) = \underline{\hspace{3cm}}$

e.) The domain of $\arctan(\ln x) = \underline{\hspace{3cm}}$