Part I - Multiple Choice

1. Given \( f(x) = x^3 \ln x \), find \( f'(e) \)
   a) \( e \)  
   b) \( 3 + 3e^2 \)  
   c) \( e^2 \)  
   d) \( 3e \)  
   e) \( 4e^2 \)

2. The population of a bacteria colony triples every 5 hours. If the population follows an exponential growth model, find \( k \).
   a) \( \ln 2 \)  
   b) not enough information  
   c) \( \ln 3 \)  
   d) \( 3 \)  
   e) \( \ln 5 \)

3. \( \lim_{x \to -\infty} \sin^{-1} \left( \frac{e^x + \sqrt{3}e^{-x}}{2e^x + 2e^{-x}} \right) \)
   a) 30  
   b) 60  
   c) \( \frac{\pi}{6} \)  
   d) \( \frac{\pi}{3} \)  
   e) more than one answer is correct

4. \( \frac{d}{dx}(\tan^{-1}(x^2)) = \)
   a) \( \frac{2x}{1 + x^4} \)  
   b) \( \frac{2}{1 + x^2} \)  
   c) \( -2x \csc(x^2) \cot(x^2) \)  
   d) \( 2x \tan^{-1}(x^2) \sec^{-1}(x^2) \)  
   e) \( \frac{2x}{1 + x^2} \)
5. \( \lim_{x \to 0} \frac{\tan(4x)}{\tan(7x)} = \)

a) \( \frac{7}{4} \)  

b) \( -\frac{7}{4} \)  

c) \( -\frac{4}{7} \)  

d) 1  

e) \( \frac{4}{7} \)

6. The graph of the DERIVATIVE of a function is shown below. On which intervals is the original function \( f \) concave down?
CIRCLE ALL CORRECT CHOICES-THERE MAY BE MORE THAN ONE!

![Graph of Derivative](image)

a) \((-\infty, -5)\)  

b) \((-5, 0)\)  

c) \((0, 4)\)  

d) \((4, \infty)\)  

e) none of these intervals

7. Circle ALL the critical values of \( f(x) = x(x-1)^{\frac{1}{2}} \)
NOTE: YOU MAY CIRCLE MORE THAN ONE CHOICE!

a) 0  

b) \( -\frac{1}{3} \)  

\( \frac{1}{4} \)  

d) \( \frac{3}{4} \)  

e) 1

8. Find the absolute maximum of \( f(x) = \sin x + \cos x \) on the interval \([0, \pi]\).
(NOTE: \( \sqrt{2} \approx 1.414 \) and \( \sqrt{3} \approx 1.73 \))

a) 1  

b) 2  

c) \( \sqrt{2} \)  

d) \( \frac{\sqrt{3} + 1}{2} \)  

e) \( \frac{\pi}{4} \)

9. The inflection points of \( f(x) = x^5 + 10x^4 \) occur at which of the following?

a) \( x = 6 \) only  

b) \( x = -6 \) only  

c) \( x = 0, x = -8 \)  

d) \( x = 0, x = -6 \)  

e) \( x = 0, x = 6 \)
10. To find the rectangle of perimeter 100 cm with the largest area, you would maximize which function?

a) \( f(x) = x(50 - x) \)  

b) \( f(x) = 2x + 2(100 - x) \)

c) \( f(x) = \frac{100}{x^2} \)  

d) \( f(x) = 50 - 2x \)

e) \( f(x) = 2x + \frac{200}{x} \)

11. Which is an antiderivative of \( f(x) = 2\sqrt{x} + \frac{1}{x^2} \)?

a) \( \frac{4}{3}x^{\frac{3}{2}} - \frac{1}{x} + C \)  

b) \( \frac{4}{3}x^{\frac{3}{2}} - \ln(x^2) + C \)

c) \( \frac{1}{\sqrt{x}} - \frac{2}{x^3} + C \)  

d) \( \frac{1}{\sqrt{x}} + \ln(x^2) + C \)

e) \( \frac{1}{\sqrt{x}} - \frac{1}{x} + C \)

12. Write \( \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \frac{1}{64} \) in summation notation.

a.) \( \sum_{n=1}^{6} \frac{(-1)^{n+1}}{2^n} \)

b.) \( \sum_{n=1}^{6} \frac{(-1)^n}{2^n} \)

c.) \( \sum_{n=1}^{6} \frac{(-1)^{n+1}}{2n} \)

d.) \( \sum_{n=1}^{6} \frac{(-1)^n}{2n} \)

e.) \( \sum_{n=1}^{6} \frac{-1}{2^n} \)

13. Find \( \int_0^5 |x - 2| \, dx \)

a.) \( \frac{9}{2} \)

b.) \( \frac{7}{2} \)

c.) \( \frac{15}{2} \)

d.) \( \frac{11}{2} \)

e.) \( \frac{13}{2} \)
14. Use the midpoint rule with $n = 4$ to approximate $\int_{1}^{3} \ln x \, dx$

a.) $\ln \frac{3465}{256}$

b.) $\frac{1}{2} \ln \frac{3465}{256}$

c.) $\frac{1}{2} \ln \frac{32}{16}$

d.) $\ln \frac{32}{16}$

e.) None of the above.

15. Find $\lim_{x \to 0} \frac{\arctan x - x}{x^3}$

a.) $-\frac{1}{3}$

b.) $-\frac{1}{2}$

c.) 0

d.) $\infty$

e.) -6

16. Suppose it is given that $\int_{1}^{3} f(x) \, dx = 7$, $\int_{0}^{2} f(x) \, dx = 3$, $\int_{2}^{3} f(x) \, dx = 13$, Find $\int_{0}^{1} f(x) \, dx$.

a.) 9

b.) 3

c.) -2

d.) 7

e.) -7
Part II - Work Out Problems

17. Find the derivative of \( f(x) = x \sec x \)

18. Find \( \lim_{x \to 0} (1 - x)^{\frac{2}{x}} \)

19. Sketch a graph with the following properties:

\[ f'(x) > 0 \text{ when } x < 1, \quad f'(x) < 0 \text{ when } x > 1 \]

\[ f''(x) > 0 \text{ when } x < -2 \text{ and } x > 2, \quad f''(x) < 0 \text{ when } -2 < x < 2 \]

\[ \lim_{x \to -\infty} f(x) = -2, \quad \lim_{x \to \infty} f(x) = 0 \]

20. Determine when the function \( f(x) = xe^{4x} \) is increasing, decreasing, concave down, and concave up.

21. Find the dimensions of the largest rectangle that can be inscribed in a circle of radius 2.

22. The acceleration of a particle is given by \( \mathbf{a}(t) = (1 + e^t)i + (\cos t)j \). If the initial velocity is \( \mathbf{i} \) and the initial position is \( \mathbf{j} \), find the position of the particle at any time \( t \).

23. A certain object cools at a rate (in °F/min) of \( \frac{1}{2} \) of the difference between its temperature and that of the surrounding air. If a room is kept at 75 °F and the initial temperature of this object is 98 °F, find an expression of the temperature of the object \( t \) minutes later.

24. By introducing a deadly chemical, the population of a particular bacteria culture is decreasing over time. Suppose it has been determined that the rate of change of the population at time \( t \) minutes is \( \frac{1}{3} \) of the population. If the initial size of the population is 1000 bacteria, how many bacteria are present after 3 minutes?

25. Find the following:

a.) \( \arccos\left(\frac{1}{2}\right) = \)

b.) \( \sin(\arccos\left(-\frac{4}{5}\right)) = \)

c.) \( \arcsin(\sin\left(\frac{5\pi}{4}\right)) = \)

d.) The domain of \( \arccos(4x - 5) = \)

e.) The domain of \( \arctan(\ln x) = \)