Spring 2020 Math 152

Week in Review XI

courtesy: David J. Manuel

(covering 11.8 and 11.9)

1 Section 11.8

1. Find the radius and interval of convergence for the following series:

   (a) $\displaystyle\sum_{n=1}^{\infty} x^n$

   (b) $\displaystyle\sum_{n=1}^{\infty} n!x^n$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{\sqrt{n+1}\,6^n}$

   (e) $\displaystyle\sum_{n=0}^{\infty} \frac{(3x-2)^n}{n^2+4}$

2. If the series $\displaystyle\sum_{n=1}^{\infty} c_n(x-6)^n$ has radius of convergence $r=3$, find the values of x for which we know the series is convergent.

3. The radius of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n(x-1)^n}{\sqrt{n}}$ is $\dfrac{1}{3}$. Find the interval of convergence.

2 Section 11.9

1. Given $f(x)=\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$, find a power series for $f'(x)$ and $\displaystyle\int f(x)\,dx$. What is $f(x)$?

2. Express the following functions as power series, and state the radius of convergence.

   (a) $f(x)=\arctan x$

   (b) $f(x)=\dfrac{x^2}{(1+x)^2}$

   (c) $f(x)=\dfrac{x}{3-x}$

3. Find the sum of $\displaystyle\sum_{n=1}^{\infty} nx^n$.

4. If $g(x)=x+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{4}x^4+\cdots+\frac{1}{n}x^n+\cdots$, find $g\left(\dfrac{1}{2}\right)$