## WIR 11 152

Monday, April 13, 2020

4:55 PM

1 Section 11.8 Power Series  $\sum_{n=0}^{\infty} c_n (x-a)^n$  a = center  $c_n = coefficients$ 

Interval of Cornergence = values of x which make series conege

- 1) radius of convergence -> ratio test

  1x-a1 < R <- radius of com
- 2) test endpoints by substituting into series for X

1. Find the radius and interval of convergence for the following series: 
$$Y^n = (x-6)^n = 0$$

(a)  $\sum_{n=1}^{\infty} x^n = x^n$ 

Restro Tost:  $|x| = |x| = |x| = |x|$ 

(b)  $|x| = |x| = |x| = |x|$ 

(c)  $|x| = |x| = |x| = |x|$ 

(c)  $|x| = |x|$ 

(d)  $|x| = |x|$ 

(example of convergence is  $|x| = |x|$ 

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(c)  $|x| = |x|$ 

(d)  $|x| = |x|$ 

(example of convergence is  $|x| = |x|$ 

(example of convergence is  $|x| = |x|$ 

(for  $|x| = |x|$ 

(g)  $|x| = |x|$ 

(horizontal property is  $|x| = |x|$ 

(horizont

(b) 
$$\sum_{n=1}^{\infty} n! x^n$$
  $Q_n = n! \times n!$ 

Rato Test:  $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{(n+1)!}{n!} \times n! \right| \times n!$ 

$$= \lim_{n \to \infty} \frac{(n+1)!}{(n+1)!} \left| \frac{x^{n+1}}{x^n} \right|$$

$$= \lim_{n \to \infty} \frac{(n+1)!}{$$

(c) 
$$\sum_{n=1}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$
  $a_n = \frac{x^{2n+1}}{(2n+1)!}$ 

Ratio Test:  $\lim_{n \to \infty} \left| \frac{a_n u}{a_n} \right| = \lim_{n \to \infty} \left| \frac{x^{2(n+1)+1}}{(2(n+1)+1)!} \cdot \frac{(2n+1)!}{x^{2n+1}} \right|^{\frac{2}{n+1}}$ 

$$= \lim_{n \to \infty} \left| \frac{x^{2n+3}}{x^{2n+1}} \right| \cdot \frac{(2n+1)!}{(2n+3)!}$$

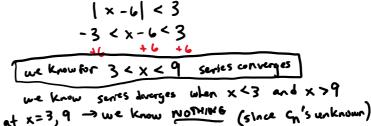
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$$R_{ahb} = \frac{1}{164} \left( \frac{2}{164} \right)^{\frac{1}{164}} = \frac{1}{164} \left( \frac{2}{164} \right)^{\frac{1}{16$$

2. If the series  $\sum_{n=1}^{\infty} c_n(x-6)^n$  has radius of convergence r=3, find the values of x for which we know the series is convergent.



3. The radius of convergence of the series  $\sum_{n=1}^{\infty} \frac{(-3)^n (x-1)^n}{\sqrt{n}}$  is  $\frac{1}{3}$ . Find the interval of convergence.

Section 11.9 Functions as Power series

Key: Geometric Series 
$$\sum_{n=0}^{\infty} ar^n = \frac{\alpha}{1-r} \frac{x/x^2/x^3/e^{\frac{1}{4}r}}{x^3/e^{\frac{1}{4}r}}$$

How?  $r = x$ :  $\frac{\alpha}{1-x}$ 

$$\frac{d}{dx} \left(\frac{\alpha}{1-x}\right) = -\alpha \left(1-x\right)^{-2} \left(-1\right) = \frac{\alpha}{(1-x)^2} = \frac{1}{dx} \sum_{n=0}^{\infty} ax^n = \sum_{n=1}^{\infty} \frac{nax^{n-1}}{ax^{n-1}}$$

$$S\left(\frac{\alpha}{1-x}\right) dx = -\alpha \ln|1-x| + C = S\left(\sum_{n=0}^{\infty} ax^n\right) dx$$

1. Given 
$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$
, find a power series for  $f'(x)$  and  $\int f(x) dx$ . What is  $f(x)$ ?

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \frac{1}{0!} + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x}{4!} + \dots$$

$$= \frac{1}{1} + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$= \frac{1}{1} + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \dots + C - 1$$

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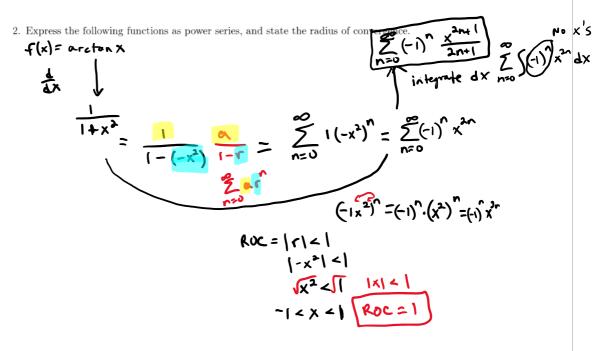
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$$= \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \dots + C - 1$$

$$= \frac{1}{1} + \frac{$$



(a)  $f(x) = \arctan x$ 

(b) 
$$f(x) = \frac{x^2}{(1+x)^2} = \frac{x^2}{(1+x)^2}$$

$$\frac{x^2}{(1+x)^2}$$

(c) 
$$f(x) = \frac{x}{3}$$
 if this was a "1" we'd have it

$$\frac{x}{3-x} \stackrel{\div 3}{:3}$$

$$= \frac{x}{3} \frac{1-x}{3} \frac{1-x}{3}$$

$$= \frac{x}{3} \frac{x}{3} \cdot \frac{x}{3} \cdot \frac{x}{3}$$

$$= \frac{x}{3} \frac{x}{3} \cdot \frac{x}{3} \cdot \frac{x}{3}$$

$$= \frac{x}{3} \frac{x}{3} \cdot \frac{x}{3} \cdot \frac{x}{3}$$

$$= \frac{x}{3} \cdot \frac{x}{3} \cdot \frac{x}{3}$$

3. Find the sum of 
$$\sum_{n=1}^{\infty} nx^n$$
.

$$x \sum_{n=1}^{\infty} nx^{n-1} = x \left( 1 + 2x^2 + 3x^3 + 4x^4 + \dots \right)$$

$$= x \left( 1 + 2x + 3x^2 + 4x^3 + \dots \right)$$

$$= x \cdot \sum_{n=1}^{\infty} \frac{d}{dx} (x^n)$$

$$= x \cdot \frac{d}{dx} \sum_{n=0}^{\infty} x^n$$

$$= x \cdot \frac{d}{dx} \left( \frac{1}{1-x} \right)$$

$$= x \cdot (+1)(1-x)^{-2}(+1)$$

$$= \frac{x}{(1-x)^2}$$

4. If 
$$g(x) = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \dots + \frac{1}{n}x^n + \dots$$
, find  $g\left(\frac{1}{2}\right)$ 

$$g(x) = \sum_{n=0}^{\infty} \frac{x^n}{n+1} \qquad g(x) = -\ln(1)x^n + \dots$$

$$g(x) = -\ln(1)x^n + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \dots + \frac{1}{n}x^n + \dots$$
, find  $g\left(\frac{1}{2}\right)$ 

if  $x = 0$ ,  $0 = -\ln(1)x^n + \dots$ 

$$g(x) = -\ln(1)x^n + \dots$$

$$g(x) = -\ln$$