ch 0: Review

Important things in calculus that you should already be familiar with:
(Note: try the examples below on your own. The answers are worked out on the following pages.)

I. Solving Equations and Inequalities

Example: \( x^2 - 2x - 8 \geq 0 \)

II. Functions/Composition of Functions

Example: If \( f(x) = \frac{1}{x - 1} \) and \( g(x) = \frac{x - 1}{x + 1} \), find and simplify:

a) \( \frac{f(x + h) - f(x)}{h} \)  

b) \( f(g(x)) \)  
c) \( g(f(x)) \)

III. Trigonometry: Please read Appendix D (pp A23-32 in text) for key trig values, identities, and graphs you WILL use this semester!

Example: Solve for \( x \): \( \sin(2x) = \cos x \)
Solutions to Examples:

**Example I**: \( x^2 - 2x - 8 \geq 0 \) (find the critical values first):

\[
x^2 - 2x - 8 = 0
\]
\[
(x - 4)(x + 2) = 0
\]
\[
x = 4 \text{ or } x = -2
\]

The critical values split the number line into 3 intervals: \( x < -2 \), \( -2 < x < 4 \), and \( x > 4 \). Use a sign chart or a number line to determine which intervals are in the solution and which are not:

<table>
<thead>
<tr>
<th>Interval</th>
<th>x-4</th>
<th>x+2</th>
<th>(x-4)(x+2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x &lt; -2 )</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>( -2 &lt; x &lt; 4 )</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( x &gt; 4 )</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Solution to the inequality is \((-\infty, -2] \cup [4, \infty)\)

**Example II**: \( f(x) = \frac{1}{x - 1} \), \( g(x) = \frac{x - 1}{x + 1} \)

a. \[
f(x + h) - f(x) = \frac{1}{x + h - 1} - \frac{1}{x - 1}
\]
\[
= \frac{1}{h} \left( \frac{(x - 1) - (x + h - 1)}{(x + h - 1)(x - 1)} \right)
\]
\[
= \frac{1}{h} \left( \frac{-h}{(x + h - 1)(x - 1)} \right) = \frac{-1}{(x + h - 1)(x - 1)}
\]

b. \[
f(g(x)) = f\left(\frac{x - 1}{x + 1}\right) = \frac{1}{\frac{x - 1}{x + 1} - 1}
\]
\[
= \frac{x + 1}{x - 1 - (x + 1)} = -\frac{x - 1}{2}; \ x \neq -1
\]

c. \[
g(f(x)) = g\left(\frac{1}{x - 1}\right) = \frac{\frac{1}{x - 1} - 1}{\frac{1}{x - 1} + 1}
\]
\[
= \frac{1 - (x - 1)}{1 + (x - 1)} = \frac{2 - x}{x}; \ x \neq 1
Example III: Solve $\sin(2x) = \cos x$ - Remember identities!!!

\[
2 \sin x \cos x = \cos x
\]

\[
2 \sin x \cos x - \cos x = 0
\]

\[
\cos x(2 \sin x - 1) = 0
\]

\[
\cos x = 0 \text{ or } 2 \sin x - 1 = 0 \rightarrow \sin x = \frac{1}{2}
\]

\[
x = \frac{\pi}{2} + 2\pi n \text{ or } x = \frac{\pi}{6} + 2\pi n; \quad x = \frac{5\pi}{6} + 2\pi n
\]

On Your Own: 0.1 #3, 4, 15, 17, 19, 21, 55, 57, 71, 87, 89, 91, 93, 99, 101; 0.2 #5, 7, 9, 13, 17; App D #1, 3, 5, 7, 9, 11, 23-28, 29, 33, 45, 47, 63, 65, 67, 71; Maplets #3, 4, 16, 24, 25 (see Syllabus for details)