1. Given the points \( A(1, 2, 3), B(1, 0, 1), \) and \( C(2, 2, 2) \) find each of the following:

   (a) the length of \( AB = \sqrt{(1-1)^2 + (2-0)^2 + (3-1)^2} = \sqrt{8} \)

   (b) \( \angle ABC \) \( \frac{BA}{BC} = \langle 0, 2, 2 \rangle, \frac{BC}{BC} = \langle 1, 2, 1 \rangle, \) so

   \[ \cos ABC = \frac{\frac{BA}{BC} \cdot \frac{BC}{BC}}{|\frac{BA}{BC}| |\frac{BC}{BC}|} = \frac{0 + 4 + 2}{\sqrt{0 + 4 + 4\sqrt{1 + 4 + 1}} = \frac{3}{2}, \text{ or } \angle BAC = 30^\circ \]

   (c) a vector orthogonal to the plane containing these points. \( \frac{AB}{AB} \) and \( \frac{AC}{AC} \) lie in the plane, so \( \frac{n}{n} = \frac{BA}{BA} \times \frac{BC}{BC} = \left| \begin{array}{ccc} i & j & k \\ 0 & 2 & 2 \\ 1 & 2 & 1 \end{array} \right| = \langle -2, 2, -2 \rangle. \)

2. Given the lines \( r(t) = \langle 2t, 2-3t, -2t-1 \rangle \) and \( x = 2s-4, y = s-2, z = 8-4s: \)

   (a) Show the lines intersect. Find the point of intersection and the angle at which the lines intersect. \( \text{If they intersect, we must have} \)

   \( 2t = 2s-4, \quad 2-3t = s-2, \quad -2t-1 = 8-4s. \) Solving the first 2 equations yields \( t = \frac{1}{2}, \quad s = \frac{5}{2}, \) which also satisfy the third equation (MUST show!). Therefore, the intersection is \( r \left( \frac{1}{2} \right) = \left( 1, \frac{1}{2}, -2 \right). \)

   (b) Find the equation of the plane containing these lines. \( \text{The direction vectors} \)

   lie in the plane, so \( \frac{n}{n} = \frac{AB}{AB} \times \frac{AC}{AC} = \left| \begin{array}{ccc} i & j & k \\ 2 & -3 & 2 \\ 2 & 1 & -4 \end{array} \right| = \langle 14, 4, 8 \rangle. \)

   Using either \( \frac{n}{n} \) or the intersection as our point, we obtain \( 14x + 4y + 8z = 0. \)

3. (15 points) Given the surface \( z = 3x^2 + y^2: \)

   (a) Describe the traces in the planes \( x = k, \ y = k, \ z = k \). \( x = k \) yields parabolas, \( y = k \) yields parabolas, and \( z = k \) yields ellipses

   (b) Identify the surface (include name and any axes). \( \text{An elliptic paraboloid along the z-axis.} \)

   (c) Find the equation of the plane tangent to this surface when \( x = 2, \ y = 1. \) \( \frac{n}{n} = \langle f_x, -f_y, 1 \rangle = \langle -6(2), -2(1), 1 \rangle. \) When \( x = 2, \ y = 1, \ z = 13, \) so the equation of the plane is \( -12x - 2y + z = -13. \)

4. Given \( r(t) = \langle t, 4\sin 2t, 2\cos t \rangle, \) find the unit tangent vector \( T(t) \) at the point

   where \( t = \frac{\pi}{6}. \) \( \frac{r''}{r''}(t) = \langle 1, 8\cos 2t, -2\sin t \rangle, \) so \( \frac{r''}{r''} \left( \frac{\pi}{6} \right) = \langle 1, 4, -1 \rangle. \) Therefore, \( T \left( \frac{\pi}{6} \right) = \langle \frac{1}{\sqrt{18}}, \frac{4}{\sqrt{18}}, -\frac{1}{\sqrt{18}} \rangle. \)
5. (Approximate answers acceptable) The radius and height of a cylindrical can are measured to be 75cm and 60cm respectively, with possible error of ±0.5 cm for the radius and ±0.2 cm for the height. Use differentials to estimate the largest possible error in calculating the surface area of the can. Recall \( SA = 2\pi r^2 + 2\pi rh \). \( d\text{SA} = \frac{\partial \text{SA}}{\partial r} dr + \frac{\partial \text{SA}}{\partial h} dh = (4\pi r + 2\pi h)dr + (2\pi r)dh \). Substituting the above numbers yields \( d\text{SA} = 240\pi \text{cm}^2 \).

6. Given the surface \( g(x, y, z) = y\tan^{-1}\left(\frac{x}{z}\right) \):

(a) Find the directional derivative of \( g \) at the point \((1, 2, -2)\) in the direction of the vector \( \vec{i} + \vec{j} - \vec{k} \). \( \nabla g = \nabla g \cdot \vec{u} \).

\[
\nabla g = \langle y \frac{1}{1 + \left(\frac{x}{z}\right)^2} \cdot \tan^{-1}\left(\frac{x}{z}\right), y \frac{1}{1 + \left(\frac{x}{z}\right)^2} \cdot \left(-\frac{x}{z^2}\right) \rangle.
\]

Evaluate at the given point: \( \nabla g = \langle -\frac{1}{4}, -\frac{\pi}{4}, -\frac{1}{4} \rangle \). \( \vec{u} \) is a unit vector in the direction of the given vector: \( \vec{u} = \langle -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \rangle \). Therefore,

\[
D_{\vec{u}}g = -\frac{1}{4\sqrt{3}} - \frac{\pi}{4\sqrt{3}} + \frac{1}{4\sqrt{3}} = -\frac{\pi}{4\sqrt{3}}
\]

(b) Find the maximum rate of change in \( g \) at the point \((1, 2, -2)\) and the direction in which it occurs. The maximum rate of change is

\[
| \nabla g | = \sqrt{\frac{1}{16} + \frac{\pi^2}{16} + \frac{1}{16}} = \sqrt{\frac{\pi^2 + 2}{16}}.
\]

The direction it occurs is the direction of \( \nabla g = \langle -\frac{1}{4}, -\frac{\pi}{4}, -\frac{1}{4} \rangle \) (which was accepted in the text and therefore on the exam, or you make the unit vector \( \frac{\nabla g}{| \nabla g |} \)).

7. Find and classify all critical points of the function \( f(x, y) = x^4 + y^4 - 4xy + 1 \) This is example 3 in the text (p776). The answer is \((0, 0)\) is a saddle point, \((1, 1)\) and \((-1, -1)\) are relative minimums.