Take-home quiz #2: 11.4-11.5

Directions: Show all appropriate work in the space provided for each problem. Partial credit will be given for appropriate work. Calculators ARE allowed, but sufficient analytic work must be shown to obtain full credit.

1. (3 pts) Find the equation of the plane which contains the point (1, 2, 0) and the line \( r(t) = \langle 3 - 2t, 2t, 2 \rangle \).

Let \( a \) be a vector from the given point to \( r_0 (3, 0, 2) \) on the line. \( a = \langle 2, -2, 2 \rangle \).

Let \( b = v = \langle -2, 2, 0 \rangle \) be the direction vector of the line.

Then \( n = a \times b = \begin{vmatrix} i & j & k \\ 2 & -2 & 2 \\ -2 & 2 & 0 \end{vmatrix} = \langle -4, -4, 0 \rangle \) is normal to the plane, so the equation of the plane is \( -4x - 4y = 12 \).

2. (3 pts) Given the surface \(-4x^2 + y^2 - z^2 = 16\):

   (a) Describe the traces of the surface in the plane \( y = k \) (include a domain for \( k \)).

   Setting \( y = k \) in the equation above yields \(-4x^2 + k^2 - z^2 = 16\), or \( 4x^2 + z^2 = k^2 - 16 \). Traces are ellipses provided \( |k| \geq 4 \).

   (b) Describe the surface. Include the name and any axes in your description.

   The surface is a hyperboloid of 2 sheets along the \( y \)-axis.

3. (4 pts)

   (a) Show that the line \( r(t) = \langle 2 - t, 3 + 2t, 3 + 4t \rangle \) does NOT intersect the plane \( 4x - 6y + 4z = 0 \).

   If they intersect, they have the same \( x, y, \) and \( z \) values at that point, so we would have \( 4(2 - t) - 6(3 + 2t) + 4(3 + 4t) = 0 \).

   Expanding the left yields \( 8 - 4t - 18 - 12t + 12 + 16t = 0 \), or \( 2t = 0 \), which has no solution. Hence, the line and plane do not intersect.

   (b) Find the distance between the line and the plane.

   Let \( a \) be a vector from a point on the line \( r_0 = (2, 3, 3) \) to a point on the plane ((0, 0, 0) works nicely). So \( a = (2, 3, 3) \).

   The vector \( n = \langle 4, -6, 4 \rangle \) is orthogonal to the plane, so the distance between them is \( \| \text{comp}_n a \| = \left| \frac{a \cdot n}{|n|} \right| = \frac{8 - 18 + 12}{\sqrt{16 + 36 + 16}} = \frac{2}{\sqrt{68}} \).