Take-home quiz #3: 11.6-12.3

Directions: Show all appropriate work in the space provided for each problem. Partial credit will be given for appropriate work. Calculators ARE allowed, but exact answers are required and sufficient analytic work must be shown to obtain full credit.

1. (4 pts)
   (a) Find a unit tangent vector to the curve \( \mathbf{r}(t) = \langle t, 4 - t, 9 + t^2 \rangle \) at the point \((2, 2, 13)\).
   \[
   \mathbf{r}'(t) = \langle 1, -1, 2t \rangle, \quad \text{and} \quad \mathbf{r}'(2) = \langle 1, -1, 4 \rangle,
   \]
   so \( T(2) = \frac{\mathbf{r}'(2)}{|\mathbf{r}'(2)|} = \left\langle \frac{1}{\sqrt{18}}, -\frac{1}{\sqrt{18}}, \frac{4}{\sqrt{18}} \right\rangle. \)

   (b) Find the point(s) where the curve \( \mathbf{r} \) above intersects the plane \( 9x - 2y + z = 7 \).
   Intersection means the same \( x, y, \) and \( z \), so at such a point we have \( 9(t) - 2(4 - t) + (9 + t^2) = 7 \).
   Expand and solve for \( t \):
   \[
   9t - 8 + 2t + 9 + t^2 = 7, \quad t^2 + 11t - 6 = 0,
   \]
   so \( t = \frac{-11 \pm \sqrt{121 + 24}}{2} \approx 0.5208, -11.5208 \)
   Therefore, the points of intersection are \( \mathbf{r}(0.5208) = \langle 0.5208, 3.4792, 9.2712 \rangle \) and \( \mathbf{r}(-11.5208) = \langle -11.5208, 15.5208, 141.7288 \rangle \)

2. (3 pts) Given \( f(x, y) = xe^{xy} \), find \( f_x(-2, 3) \) and \( f_y(-2, 3) \).
   \[
   f_x(x, y) = e^{xy} + xye^{xy}, \quad \text{so} \quad f_x(-2, 3) = e^{-6} - 6e^{-6} = -5e^{-6}
   \]
   \[
   f_y(x, y) = x^2e^{xy}, \quad \text{so} \quad f_y(-2, 3) = 4e^{-6}. \)

3. (3 pts) Determine whether the function \( u = \sin(x - 3t) + \ln(x + 3t) \) is a solution to the wave equation \( u_{tt} = 9u_{xx} \).
   Differentiate and substitute:
   \[
   u_t = -3 \cos(x - 3t) + \frac{3}{x + 3t}, \quad u_{tt} = -9 \sin(x - 3t) - \frac{9}{(x + 3t)^2},
   \]
   \[
   u_x = \cos(x - 3t) + \frac{1}{x + 3t}, \quad u_{xx} = -\sin(x - 3t) - \frac{1}{(x + 3t)^2} \quad \text{so} \quad u_{tt} = 9u_{xx}. \)