

Review for EXAM # 2

MATH 142

Drost-Fall 2006

3.1 Limits

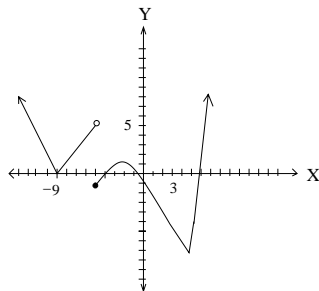
- 1) Find $\lim_{x \rightarrow 2} \frac{1}{x-2}$
- 2) Given $f(x) = \begin{cases} x^3 - x + 2, & \text{if } x < 2 \\ x^4 - 3x^2 + 1, & \text{if } x \geq 2 \end{cases}$
 - (a) Find $\lim_{x \rightarrow 2^-} f(x)$
 - (b) Find $\lim_{x \rightarrow 2^+} f(x)$
- 3) Find $\lim_{x \rightarrow 1} \frac{\sqrt{x}}{x-3} \cdot \frac{x+2}{x^2+1}$

Find each of the following limits:

- 4) $\lim_{x \rightarrow 3} \frac{x^2 - 3x}{x^2 - 9}$
- 5) $\lim_{x \rightarrow -3} \frac{x^3 + 27}{x + 3}$
- 6) Given $f(x) = \begin{cases} x^2 + 3, & \text{if } x < 2 \\ |x + 5|, & \text{if } x > 2 \end{cases}$
 - (a) Find $\lim_{x \rightarrow 2^-} f(x)$
 - (b) Find $\lim_{x \rightarrow 2^+} f(x)$
 - (c) Find $\lim_{x \rightarrow 2} f(x)$
- 7) $\lim_{x \rightarrow 16} \frac{x - 16}{\sqrt{x} - 4}$
- 8) $\lim_{x \rightarrow 0} |x|$

3.2 Continuity and Nondifferentiability

- 9) Where is the function $f(x) = \begin{cases} x^2 + 6, & \text{if } x < 2 \\ |x - 5|, & \text{if } x > 2 \end{cases}$ not continuous?
- 10) Where is the function described in the previous problem, not differentiable?
- 11) Where is the function $f(x) = \frac{x^2 - 3x + 2}{x^2 + x - 6}$ not continuous?
- 12) Where is the function described in the previous problem is not differentiable?
- 13) Where is the function pictured below not differentiable?



State where (if any) the following functions are not continuous:

$$14) g(x) = \frac{x^2 - 1}{x^2 + 7}$$
$$15) h(x) = \frac{7}{x^2 + x}$$

3.3 The Derivative

- 16) Find the average change in revenue if production is changed from 200 items to 500 items when $R(x) = x^2 - 30x + 45$.
- 17) An object moves along the y -axis measured in feet, so that $y = 2x\sqrt{x} + 9$, where x represents time in seconds. After finding the instantaneous velocity function, find the velocity at 1 minute.
- 18) Find the slope of the tangent line to $f(x) = \sqrt{x+3}$ at $x = 1$.
- 19) The following table gives the population in hundreds of thousands over a given period of time. Find the **average rate of change** of the population with respect to time from 1985 to 1988.

Year	'85	'86	'87	'88	'89	'90
Population	3.2	3.6	3.8	3.9	4.2	4.3

- 20) The profit in millions of dollars is given by $P = -x^2 + 2x - 8$ where x is the number of cars sold in thousands. Find the instantaneous rate of change when 4,000 cars are sold.
- 21) $f(x) = 2x^2 - x$; find $f'(x)$ using the limit definition.
- 22) $f(x) = \frac{1}{x+2}$, $x \neq -2$; find $f'(x)$ using the limit definition.
- 23) $f(x) = \sqrt{x+5}$; find $f'(x)$ using the limit definition.
- 24) Given a graph of $f(x)$, what does f' look like?

3.4 Derivatives of Constants, Powers and Sums

- 25) $f(x) = 4x^{\frac{2}{3}}$, find the derivative.
- 26) $g(x) = 5e^2$, find the derivative.
- 27) $F(x) = \frac{5x^2 - 4x + 15}{3x}$, find the derivative.
- 28) $G(x) = 6\sqrt[3]{x} + \pi$, find the derivative.
- 29) The number of dollars spent on advertising for a new CD influences the number of CDs consumers purchase. If x is the number of dollars spent, in thousands, then $N(x)$, measured in hundreds, is the number of CDs sold, as given by $N(x) = 2500 - \frac{520}{x}$
 - (a) Compute $N'(x)$.
 - (b) Compute $N(10)$ and interpret.
 - (c) Compute $N'(10)$ and interpret.

3.5 Product and Quotient Rule

30) $g(x) = (x^{\frac{2}{3}} + 2x - 5)(x^{\frac{1}{2}} - 7x^{-2} + 6)$,
find the derivative.

31) $h(x) = \frac{2x^3 - 1}{x^2 + 3}$, find $h'(x)$.

32) Given $f(w) = \frac{5\sqrt[3]{w}}{w - 2}$, find $\frac{df}{dw}$.

3.6 The Chain Rule

33) $f(x) = (x^2 + 1)^5$ Find $f'(x)$

34) Find the values of x where the tangent line is horizontal for $f(x) = \frac{x^3}{(4 - x)^5}$

3.7 Marginal Analysis

The Wicket Ewok Factory has costs described by the function, $C(x) = 15x + 4\sqrt{x} + 80$ and charges \$5 more than the number ordered. Let x be the number of stuffed animals made and sold.

35) Find the marginal cost.

36) Find the marginal revenue.

37) Find the marginal profit.

38) Find the quantity they need to make and sell to break even.



Hint: Use your calculator to find the intersection of 2 functions.

39) Given $R(x) = 4x^2 - \frac{1}{2}x + 8$, find the marginal average revenue.

40) Given $C(x) = 250 - 5x$, find the average cost for $x = 8$.

41) Solar Energy INC, creates sidewalk lanterns which run on solar energy. They have determined a price demand function for their product of $x = 2500 - 100p$. They have fixed costs of \$504, and variable costs of \$12 per item.

- Find the domain of the price-demand function.
- Find the cost function.
- Find $R(x)$, the revenue function in terms of the quantity produced and find its domain.
- Find the quantity they should produce and sell to maximize profit.

4.1 First Derivative Tests and Graphs

42) Find the critical values of

$$g(x) = \frac{x^2 - 4x}{x^2 - 6x + 8}$$

43) Sketch the graph described below:
domain $\mathbb{R}, x \neq 2$
intercepts $(0, 4), (4, 0)$

horizontal asym $y = 2$

vertical asym $x = 2$

$\searrow (2, 4), \nearrow (-\infty, 2), (4, \infty)$

$\cup (-\infty, 2), (2, 6); \cap (6, \infty)$

4.2 Second Derivative

44) Find where $f(x)$ is increasing/decreasing, concave up/down, relative extrema, and points of inflection for $f(x) = x + \frac{5}{x}$.

4.3 Graphing Rational Functions

45) $f(x) = \frac{-10}{x^2 - 1}, f'(x) = \frac{20x}{(x^2 - 1)^2}$,

$$f''(x) = \frac{-20 - 60x^2}{(x^2 - 1)^3}$$

a) find intercept(s)

b) find asymptote(s)

c) find the interval(s) over which the function is increasing or decreasing.

d) find the interval(s) over which the function is concave up/down.

e) graph the function

46) $\lim_{x \rightarrow \infty} \frac{3x^2 - 2x + 5}{9x^2 - x + 1}$

47) $\lim_{x \rightarrow \infty} \frac{2x^3 + 1}{5x^2 - 4}$

48) $\lim_{x \rightarrow \infty} \frac{6x^2 + 1}{5x^3 + 7}$

49) $\lim_{x \rightarrow -\infty} \frac{-4x^2 + 8}{x + 3}$

4.4 Absolute Extrema

50) Determine the absolute extrema:

(a) $f(x) = x^3 + 4x^2 - 3x - 6$ over $[-2, 2]$

(b) $f(x) = \frac{x^2}{x - 2}$ over $(2, 6)$

51) Given $f(x) = \frac{x - 1}{x^2}$, find the absolute extrema on the interval $(-\infty, \infty)$

4.5 Optimization

52) Find the *instantaneous rate of change of concentration*, given the concentration of drug "Headache Cure-All" in the bloodstream at time t is $C(t) = 150(1 - 4t^3)^2$, $t \geq 0$, where $C(t)$ is the concentration of the drug in the bloodstream in micrograms per milliliter, and t is measured in minutes.

53) The Digital Pet Company has determined that its daily cost in dollars, for producing x virtual pets is given by

$$C(x) = 150 + 3x + \frac{1}{6}x^2, \quad 0 \leq x \leq 200.$$

(a) Find the average cost function.

(b) How many virtual pets should they make to minimize the average costs?

- 54) Find two numbers whose sum is 20 for which the sum of the squares is a minimum.
- 55) The local travel agency offers one week in Hawaii, for \$840 if 20 sign up, and discounts of \$10 per person for each additional person over the minimum of 15. If the 747 holds 300, what price should they charge to maximize revenue?

ANSWERS

1. dne

2a. 8

2b. 5

3. $\frac{-3}{4}$

4. $\frac{1}{2}$

5. 27

6a. 7; b. 7; c. 7

7. 8

8. 0

9. at $x = 2$

10. at $x = 2, 5$

11. at $x = -3, 2$

12. at $x = -3, 2$

13. at $x = -9, -5, 5$

14. continuous everywhere

15. at $x = 0, -1$

16. 670

17. 23.8ft/sec

18. $m = \frac{1}{4}$

19. 23, 333 people/year

20. profit is decreasing \$6,000/car

21. $4x - 1$

22. $\frac{-1}{(x+2)^2}$

23. $\frac{1}{2\sqrt{x+5}}$

25. $f'(x) = \frac{8}{3}x^{-\frac{1}{3}}$

26. $g' = 0$

27. $F'(x) = \frac{5}{3} - 5x^{-2}$

28. $G'(x) = 2x^{-\frac{2}{3}}$

29a. $N'(x) = \frac{520}{x^2}$

29b. $N(10) = 2448$; When \$10,000 is spent on advertising, 244,800 CD's were sold.

29c. $N'(10) = 5.2$; When the amount spent on advertising is \$10,000, an additional \$1,000 will result in 520 additional CD's being sold.

30. $g'(x) = (x^{\frac{2}{3}} + 2x - 5)(\frac{1}{2}x^{-\frac{1}{2}} + 14x^{-3}) + (x^{\frac{1}{2}} - 7x^{-2} + 6)(\frac{2}{3}x^{-\frac{1}{3}} + 2)$

31. $h'(x) = \frac{2x^4 + 18x^2 + 2x}{(x^2 + 3)^2}$

32. $\frac{-10(w+1)}{3w^{\frac{2}{3}}(w-2)^2}$

33. $f'(x) = 5(x^2 + 1)^4(2x)$

34. $x = 0, -6$

35. $MC = 15 + \frac{2}{\sqrt{x}}$

36. $MR = 5 + 2x$

37. $MP = 2x - 10 - \frac{2}{\sqrt{x}}$

38. 16 stuffed animals

39. $MAR = 4 - \frac{8}{x^2}$

40. \$26.25

41a. $0 \leq p \leq 25$, b. $C = 504 + 12x$, c. $R = 25x - 0.01x^2$, $0 \leq x \leq 2500$, d. $x = 650$

42. none

44. inc: $(-\infty, -\sqrt{5}), (\sqrt{5}, \infty)$

dec: $(-\sqrt{5}, 0), (0, \sqrt{5})$

$\cup(0, \infty), \cap(-\infty, 0)$

points of inflection: none

45. a. $(0, 10)$,

b. $x = 1, -1, y = 0$,

c. $\nearrow (0, 1), (1, \infty), \searrow (-\infty, -1), (-1, 0)$

d. $\cup(-1, 1), \cap(-\infty, -1), (1, \infty)$

46. $\frac{1}{3}$

47. ∞

48. 0

49. ∞

50. a. absolute max at $x = 2$
absolute min at $x = \frac{1}{3}$

b. absolute max: none
absolute min at $x = 4$

51. absolute max is $\frac{1}{4}$, no absolute min
52. $C'(t) = -3600t^2 + 14,400t^5$
53. a. $AC = \frac{150}{x} + 3 + \frac{1}{6}x$ b. 30 virtual pets
54. $x = 10, y = 10$
55. \$520