Review for EXAM # 3

MATH 142 Drost-Fall 2009

5.4 Curve Sketching

1) Sketch the graph of $f(x) = \dfrac{4}{4-x^2}$ and list intercepts, asymptotes, critical values, and local extrema, if they exist.

2) Given $g'' = (x-0.1)^2(x-2)(x+1)^3$, detemine where $g(x)$ is concave up/down.

3) Find all asymptotes for the function $f(x) = \dfrac{4x^2-2x+7}{2x+1}$

4) Given the cost function for Santa's Workshop is $C(x) = 2800 + 0.25x^2$, where x is the number of items produced, find the minimum average cost.

5) Draw the graph of $f(x)$ as described below:

a) domain: $\Re, x \neq 4$

b) intercept: $(0, 0)$

c) increasing: $(-2, 2)$

d) decreasing: $(-\infty, -2) \cup (2, 4) \cup (4, \infty)$

e) concave up: $(-3, 1) \cup (4, \infty)$

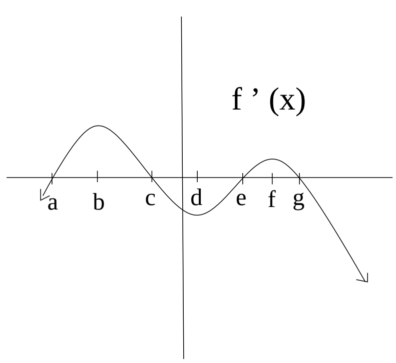
f) concave down: $(-\infty, -3) \cup (1, 4)$

g) horizontal asymptote: $y = 0$

5.5 Absolute Extrema

6) Find the x-value(s) where the absolute extremum of $f(x) = \dfrac{x^2 - 9}{x - 5}$ occur, if they exist, on the interval $[-3, 5]$, given $f\,' = \dfrac{x^2 - 10x + 9}{(x - 5)^2}$.

7) Given the graph of $f\,'(x)$, determine the intervals where $f(x)$ is increasing or decreasing.



8) Determine from the graph above of $f'(x)$ where $f(x)$ is concave up or down.

9) Find the absolute extrema for $g(x) = \dfrac{x}{\ln x}$ on the interval $[2, \infty)$.

10) Find the absolute extrema for

$$h(x) = -3x^4 + 5x^3 - 2x^2 - 6x + 12 \text{ on the interval } [-2, 5].$$

5.6 Optimization

11) When the monthly rent is \$850 all 120 apartments are filled. Each time the rent is raised \$25, 5 apartments become vacant. If the cost function to maintain the apartments is $C(x) = 85x + 2000$, find the price they should rent the apartments for, to maximize profit.

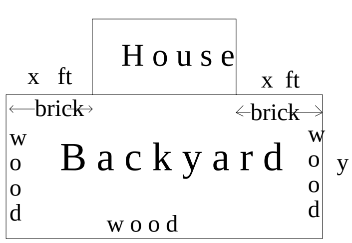
12) Poor Yoricks Coffee House at Evan's Library sells 1200 cups of coffee per day at a price of \$2.75. A market survey shows that for every 5 cent reduction in price, 40 more cups of coffee are sold. What price should Poor Yorick's charge to maximize revenue on the sale of coffee?

13) A pizza box is made out of a rectangular piece of cardboard which is 15 inches by 15 inches. Squares of equal size will be cut from each corner, and then each side is folded up to form a rectangular box. What size square should be cut from each corner to maximize the volume?

14) A pharmacy has a uniform annual demand for 1200 bottles of Trazodone. It costs \$10 to store one bottle for a year and \$50 to place an order. How many times during the year should the order be placed in order to minimize the total storage and reorder costs?

15) A fence is being built to enclose a rectangular area in the back yard using the back of the house as one part of the enclosure. The house is 80 feet wide. Find the dimensions that will maximize area if:

a) the fence attached to the house costs \$42 per linear foot, and the remaining three sides cost \$28 per linear foot, and the budget for building the fence is \$3024.



b) if 500 feet of wood fencing is available and they decide not to use brick along the street side of the fence.

6.1 Indefinite Integrals

16) $\displaystyle\int \frac{\ln\,x}{x}\,dx$

17) $\displaystyle\int \frac{1-y^3}{y}\,dy$

18) $\displaystyle\int \left(2\sqrt{x}+\frac{5}{\sqrt{x}}\right)\,dx$

19) $\displaystyle\int \frac{e^x-4x}{5}\quad dx$

20) $\displaystyle\int \frac{(1+3x)^2}{x^3}\,dx$

6.2 Integration by Substitution or the "Fix-It" Method

21) $\int 6(x+1)(2x^2+4x-5)^{\frac{3}{2}} \; dx$

22) $\int 5x(3x-18)^{\frac{1}{2}} \; dx$

23) $\int \frac{x}{\sqrt[3]{x^2-1}} \; dx$

24) $\int \frac{x^3}{\sqrt{x^4-4}} \; dx$

25) $\int_{-5}^{0} \sqrt[3]{2-x} \; dx$

6.4 The Definite Integral

26) Using Riemann Sums, find the approximate area under the curve $f(x) = 0.5x^2 - 2x + 4$ over the interval $[1, 9]$ using four rectangles: a) from the left, b) from the right, c) using midpoints.

27) In using Riemann sums, what is the height of the third rectangle, from the left, in finding the approximate area, R_5 for the function $f(x) = 0.2x^2 - x + 10$ over the interval $[5, 15]$?

28) $\int_2^5 (3x^2 - 12x + 6)\, dx$

29) $\int_1^2 \left(\star x^5 - 4\triangle x^3 + 2\clubsuit x\right)\, dx$

30) Given: $\int_a^b 4x^3\, dx = 609$ and $\int_a^4 4x^3\, dx = 240$. Find the value of b.

6.5 Fundamental Theorem of Calculus

31) $\int_1^3 \left(4 - 32x^{-3}\right)\, dx$

32) $\int_0^9 x\sqrt{9-x}\quad dx$

33) Find the **exact value** of $\int_0^{\sqrt{3}} \frac{x}{4-x^2}\quad dx$

34) The Ringin' Ringtone Company finds their marginal cost is defined by $C'(x) = 200 - 2x$ where x is the number of ringtones produced each month. Compute the increase in cost to change the production level from 50 ringtones each month to 100 ringtones per month.

35) a. Write the integral to represent the area below $f(x)$ and above the x-axis when

$$f(x) = \begin{cases} 4x+4, & x < 1 \\ 9-x^2, & x \geq 1 \end{cases}$$

b. Sketch the graph.

c. Find the area.

7.1 Area Between Curves

36) Find the area bounded by the curves $f(x) = x^2 + 1$ and $g(x) = -2x + 16$.

37) Find the area bounded by the curves $f(x) = x^3 - 7x - 6$ and $g(x) = 6x + 6$.

38) Find the useful life of the ringtone produce by Ringin' Ringtone Company, if the marginal cost is defined by $MC = 3t + 75$, where t is the number of weeks since it was installed by the user, and the marginal revenue is $MR = 200 - 2t$.

39) Find the total profit accumulated over the useful life of the ringtone.

40) Distribution of Wealth. The data in the table below, describes the distribution of wealth in a country.

x	0	0.20	0.40	0.60	0.80	1
y	0	0.15	0.28	0.55	0.68	.42

a) Use quadratic regression to find the equation of a Lorenz curve for the data.

b) Use the regression and a numerical integration routine to approximate the Gini index of income concentration.

7.2 Applications

41) Find the consumers' surplus at a price level of \$12 for the price-demand function $p = D(x) = 40 - .5x$.

42) Find the point of equilibrium for the Company XYZ which has a price-demand function of $p = D(x) = 8 - 1.5x$ and a price-supply function of $p = x^2 + 1$.

43) Find the producers' surplus at the equilibrium point for problem number 42.

44) Find the consumers' surplus at the equilibrium point for problem number 42.

45) The rate of change of the income provided by the vending machines on the first floor of Blocker is given by $f(t) = 2500e^{0.03t}$, where t is time in years since the installation of the machine. Find the total income produced the third year.

Miscellaneous Topics

46) Find any extrema indicated by the following information:

a) $f(2) = 4, f'(2) = 0.f''(2) = -1$

b) $f(5) = -3, f'(5) = 0.f''(5) = 0$

c) $f(-8) = 0, f'(-8) = 0.f''(-8) = 3$

47) Given $y' = x(x-1)^2(x+2)^3$, over what intervals is $f(x)$ increasing?

48) Find the absolute extrema over the interval $[-2, 2]$ for $f(x) = \dfrac{2x}{e^{-x}}$.