Week in Review # 15

MATH 142

Final EXAM Review

Drost-Fall 2006

1. Find the domain of the function $f(x) = \begin{cases} \dfrac{\sqrt{x-1}}{x-2} & \text{if } x < 2 \\ \sqrt{5-x} & \text{if } x > 3 \end{cases}$

2. Suppose 450 items are sold per day at a price of \$53 per item and that 650 items are sold per day at a price of \$45 per item. Find the demand function for p, assuming the demand curve to be a straight line.

3. How much money would have to be invested at $6\frac{3}{4}\%$ compounded weekly to get back \$5000 at the end of 8 years? (Express answer rounded to the nearest cent.)

4. Solve $3\log(2x+5)+6=0$

5. The price-demand function $p = \sqrt[3]{32-x}$ models the price of Gary's Gadgets. Find the elasticity of demand when the price is \$1.50. Should the price remain the same, or be raised or lowered in order to increase revenue?

6. Complete the square and put in standard form: $y = -2x^2 + 12x + 15$

7. If $p = 40 - 2.5x$, find the marginal revenue at $x = 6$

8. Estimate the cost of the 100^{th} item if $C(x) = x^2 - 50x + 275$

9. Given $f(x) = \begin{cases} e^{2\ln a} & \text{if } x < 7 \\ x-3 & \text{if } x \geq 7 \end{cases}$, for what value(s) of a is $f(x)$ continuous?

10. $\displaystyle\lim_{x\to 2} \frac{x^2-x-2}{x^2+x-6}$

11. Given $f(x) = x^3 \cdot e^{4x}$, find $f'(x)$

12. Find the derivative of $f(x) = \sqrt{x^2-4}$

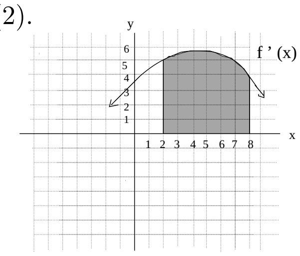
13. The function $f(x) = \dfrac{2x^2+x-1}{2x^2+23x-12}$ has a vertical asymptote at:

14. Estimate: $\displaystyle\int_2^3 \sqrt[5]{4+8x^3}\ 4x\,dx$

15. Given marginal cost defined by $50e^x + 24x - 200$ and the company has fixed costs of \$150. Find the cost function.

16. If the $\displaystyle\int_a^b 12x^3\ dx = 3840$ and $\displaystyle\int_a^4 12x^3\ dx = 720$ find the sum of $a + b$, when $a > 0$ and $a < b$.

17. Given the graph of $y = f'(x)$. Suppose $f(8) = 72$ and the area of the shaded region is 56. Find $f(2)$.



18. Simplify: $\log_3 243 - e^{2\ln 4} + \log 10$.

19. Given the supply equation $S(x) = 3x^2 + 50$ and the demand equation $D(x) = -2x^2 + 230$, find the producer's surplus.

20. Mr. Durham bought a giant screen TV for \$3800, new. It was expected to last 18 years with a scrap value of \$830. Assuming a linear depreciation, what is the set worth in five years?

21. Given $y = \frac{1}{3}x^2 - \frac{3}{4}x + 9$, locate the vertex.

22. Solve $3 \cdot 10^{2-5x} = 12$

23. Solve $\log(\log(5+x)) = 0$

24. An apartment has 80 units. When the rent is \$750 per month, all the units are rented. For each \$25 increase in rent, one apartment becomes vacant. Find the rent he should charge to maximize revenue.

25. $\displaystyle\lim_{x\to\infty} \frac{2e^x - x}{e^x + 3x}$

26. Given the following data, find the average rate of change on the interval $[3, 11]$.

x	3	5	7	9	11	15
y	9	17	21	29	31	63

27. If $C(x) = 2x^2 + 50$, find the number of items necessary to minimize the average cost per item.

28. Given $y = \dfrac{3x - e^{4x}}{\ln\ x}$, find $\dfrac{dy}{dx}$ and DO NOT SIMPLIFY.

29. Find $f''(x)$, the second derivative of $f(x) = x^2 \cdot \ln x$.

30. Given: $f(x) = \dfrac{6 - 3x + 4x^2}{18 - 2x^2}$ Find the horizontal asymptote(s).

31. Find the absolute minimum of $f(x) = x^3 - 3x^2 + 6$ on the interval $[0, 4]$.

32. Estimate $\displaystyle\int_{-2}^{1} e^{x^3}\ dx$ (to two decimal places) by finding the left-hand sums for $n = 10$.

33. Find the EXACT value of $\displaystyle\int_{-2}^{0} \sqrt{4 - x^2}\ dx$ by finding the area of the appropriate geometric region.

34. Find the average value of $f(x) = 16 - x^2$ on the interval $[0, 4]$.

35. Find the area enclosed by the curves: $y = x^2 - 6$ and $y = -x$

36. Evaluate $f(x, y) = \sqrt{8 - x - y^2}$ at $(-4, 2)$

37. Find the domain:

    a. $f(x, y) = \dfrac{y}{x - 2y}$

    b. $f(x, y) = \sqrt{8 - x + y}$

    c. $f(x, y) = 3x^2\sqrt{y}$

    d. $f(x, y) = \sqrt[3]{16 + x^2 + y}$

38. Find the surface area of closed rectangular box whose volume is 1000 ft^3 as $S(x, y)$

39. A company sells gadgets and widgets. The gadjets sell at $p = 120 - 2x - 3y$ and the widgets sell at $q = 200 - x - 5y$, where $x =$ the number of gadgets sold and $y =$ the number widgets sold. Find the revenue function, $R(x, y)$, and the value of $R(10, 20)$.

40. Maximize the revenue in the previous problem.

41. Find f_x and f_y given $f(x, y) = 3x + 4y^2 - 2xy$

42. Find f_x and f_y given $f(x, y) = \sqrt{8 - x^2 - y^2}$

43. Find f_x and f_y given $f(x, y) = x^4 \cdot e^{2xy}$

44. Find f_x and f_y given $f(x, y) = \dfrac{x}{y^2 - 1}$

45. Find the second partial derivatives of: $f(x, y) = 3x^2 + 4y^2 - 2x^2y^3$.

46. Find the second partial derivatives of: $f(x, y) = e^{x-2y}$

47. The Bollinger Processing Plant is producing palm trees, x = the number of small trees, and y= the number of pairs of large trees. The small trees sell for \$25 each, and the large trees sell for \$100 per pair. The cost of producing these trees is given by the function $C(x, y) = 5x^2 - 10xy + 10y^2$. Find the number of each they should produce to maximize profit.

48. Find where the function $f(x) = x^2 - \ln x$ is decreasing.

49. Determine the interval over which $f(x) = x^3 - 6x^2 - 15x + 10$ is concave down.

50. A video store expects to sell 10,000 copies of **Chicago** during the coming year. It costs the company 10 cents to store a video for a year, and there is a \$125 shipping and handling fee for each order. If each video costs the store \$8, what size orders should they place to minimize inventory costs?

51. $\displaystyle\int (x - 2)e^{3x^2 - 12x}\ dx$

52. Find the absolute extrema for $f(x) = 12 + 4x + \dfrac{25}{x}$ on the interval $[2, 10]$.

53. Find $f(x)$ given $f'(x) = 4e^{8x} + 2$ if $f(0) = 5$.

54. $\displaystyle\int 4^{x^2+1}(4x)\ dx$

55. $f(x) = \begin{cases} \text{6-x} & \text{if } x < -10 \\ 2x^2 + 15x - 30 & \text{if } -10 \leq x < 0 \\ -10 + 5\log_2(\frac{1}{16}) & \text{if } x = 0 \\ -(2^{5\log_2(x+2)} - 2) & \text{if } x > 0 \end{cases}$

For what interval(s) is $f(x)$ continuous?

56. Given $f(x) = x^2 - 1, g(x) = \dfrac{x}{x-3}$, and $h(x) = \sqrt{7-x}$. Find $(f \circ g \circ h)(3)$

Answers:

1. $[1, 2) \cup (3, 5]$
2. $p = -.04x + 71$
3. \$2,914.76
4. $x = -2.495$
5. .3537, raised to increase revenue
6. $y = -2(x-3)^2 + 33$
7. 10
8. \$148
9. $a = 2$
10. $\dfrac{3}{5}$
11. $f'(x) = x^2(e^{4x})(4x+3)$
12. $f'(x) = \dfrac{x}{\sqrt{x^2-4}}$
13. $x = -12$
14. 26.6031
15. $C(x) = 50e^x + 12x^2 - 200x + 100$
16. 8
17. 16
18. -10
19. \$432
20. \$2,975.
21. $\left(\dfrac{9}{8}, \dfrac{549}{64}\right)$
22. $x = \frac{1}{5}(2 - \log 4)$
23. $x = 5$
24. \$1375.
25. 2
26. 2.75
27. $x = 5$
28. $\dfrac{dy}{dx} = \dfrac{(\ln x)(3 - 4e^{4x}) - (3x - e^{4x})(\frac{1}{x})}{(\ln x)^2}$
29. $f"(x) = 3 + \ln x^2$
30. $y = -2$
31. 2
32. 1.89
33. π
34. $10\frac{2}{3}$
35. $20.8\overline{3}$
36. $2\sqrt{2}$
37. a) $y \neq \frac{x}{2}$ b) $y \geq x - 8$ c) $y \geq 0$ d) $\Re$
38. $S_A = 2000y^{-1} + 2000x^{-1} + 2xy$
39. $R(x, y) = 120x - 2x^2 - 4xy + 200y - 5y^2$ $R(10, 20) = 2200$
40. $x = 16\frac{2}{3}, y = 13\frac{1}{3}$
41. $f_x = 3 - 2y, f_y = 8y - 2x$
42. $f_x = \dfrac{-x}{\sqrt{8 - x^2 - y^2}}, f_y = \dfrac{-y}{\sqrt{8 - x^2 - y^2}}$
43. $f_x = 2x^3e^{2xy}(xy + 2), f_y = 2x^5e^{2xy}$
44. $f_x = \dfrac{1}{y^2 - 1}, f_y = \dfrac{-2xy}{(y^2-1)^2}$
45. $f_{xx} = 6 - 4y^3, f_{xy} = -12xy^2, f_{yx} = -12xy^2, f_{yy} = 8 - 12x^2y$
46. $f_{xx} = e^{x-2y}, f_{xy} = -2e^{x-2y}, f_{yx} = -2e^{x-2y}, f_{yy} = 4e^{x-2y}$
47. maximum profit with 15 small trees, and 25 large trees
48. $(0, \frac{1}{\sqrt{2}})$
49. $(-\infty, 2)$
50. 5,000 videos
51. $\dfrac{1}{6}e^{3x^2-12x} + C$
52. MAX = 54.5, MIN = 32
53. $f(x) = 0.5e^{8x} + 2x + 4.5$
54. $\dfrac{2 \cdot 4^{x^2+1}}{\ln\ 4} + C$
55. $(-\infty, -10), (-10, \infty)$
56. 3