Math142 Week In Review # 5

The Most Important Problems to Understand - This Week

1. Use the four-step process to find the derivative of $f(x) = 5 - 3x^2$.

2. Find the derivative of $f(x)$ given that $f(x+h) - f(x) = \dfrac{3}{2x+2h-5} - \dfrac{3}{2x-5}$.

3. Find the derivative using the limit definition of the derivative for $f(x) = \sqrt{4-3x}$.

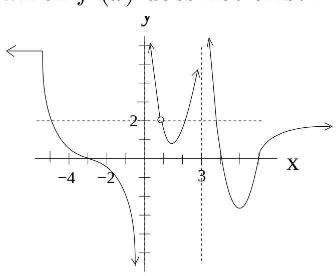
4. Using the function defined in #1 above, find the equation of the tangent to $f(x)$ at $x = -1$.

5. An object moves along the y- axis as described by $y = x^2 + 2x$, where x is the time in seconds, and y is measured in feet.

   a. Find the average velocity when x changes from 2 seconds to 5 seconds.

   b. Find the instantaneous velocity at x=2 seconds.

6. Refer to the graph of $f(x)$ below. List the values of x for which $f'(x)$ does not exist .



7. The profit in dollars from the sale of "x" Wii © consoles is given by $P(x) = 200x - 0.01x^2 - 3000$.

   a. Find the average change in profit if production changes from 7000 units to 8000 units.

   b. Find $P'(x)$ using the limit definition of the derivative.

   c. Find the instantaneous rate of change of profit at 7000 units.

   d. As the plant manager, would you keep production the same, increase, or decrease production?

8. Find the derivitive, y' if

   a. $y = x^{-4} + x^{\frac{3}{2}} - e^3$.

   b. $y = \dfrac{1}{\sqrt[3]{x^2}}$.

   c. $y = 4.2x^{-2} - \dfrac{0.5}{\sqrt[4]{x}} + 2$.

   d. $y = x^2 - 1.5x - 10\sqrt{x}$.

   e. $y = \dfrac{x^5 - 5x^3 - 2}{x^2}$.

9. How can the derivative be used to find the maximum and minimum?

10. The price-demand function and the cost function for the production of air-conditioning units is $x = 2000 - 0.25p$ and $C(x) = 60,000 + 200x$.

    a. Find the average cost of making 100 units.

    b. Find the marginal cost of making 100 units.

    c. Find the marginal average cost when $x = 100$.

    d. Find the revenue when 100 units are made and sold.

    e. Find the average revenue when 100 units are made and sold.

    f. Find the revenue of making and selling 25 units.

    g. Find the approximate revenue from the 25^{th} unit.

    h. Find the marginal average revenue function.

    i. What is the profit from making and selling 100 units?

    j. What is the marginal profit function.

    k. Find the marginal average cost function.

    l. How many should they make and sell to maximize revenue?

    m. How many should they make and sell to maximize profit?

1. $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = -6x$

2. $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \dfrac{-6}{(2x-5)^2}$

3. $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \dfrac{-3}{\sqrt{4-3x}}$

4. $y = 6x + 8$

5. a. 9ft/sec, b. 6ft/sec

6. at $x = -5$, $x = 0$, $x = 1$, $x = 3$, and $x = 6$

7. a. 50 dollars per unit

   b. $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = f'(x) = 200 - 0.02x$

   c. $P'(7000) = 60$

   d. $P(x) = 200x - 0.01x^2 - 3000$

8. a. $y' = -\dfrac{4}{x^5} + \dfrac{3}{2}\sqrt{x}$

   b. $y' = -\dfrac{2}{3x^{\frac{5}{3}}}$

c. $y' = -\frac{8.4}{x^3} + \frac{1}{8x^{\frac{5}{4}}}$

d. $y' = 2x - 1.5 - \frac{5}{\sqrt{x}}$

e. $y' = 3x^2 - 5 + \frac{4}{x^3}$

9. At a max or min, the slope of the tangent is zero. Set $f'(x) = 0$ and solve. A max or min may also occur where the derivative DNE, in the case of a corner or a cusp.

10. a. \$800

    b. \$200

    c. -6 dollars per unit

    d. $\$760,000$

    e. \$7600

    f. $\$197,500$

    g. \$7808

    h. -4

    i. $\$734,000$

    j. $MP = P' = 7800 - 8x$

    k. $MAC = -\frac{60,000}{x^2}$

    l. $1,000$ items

    m. 975 items