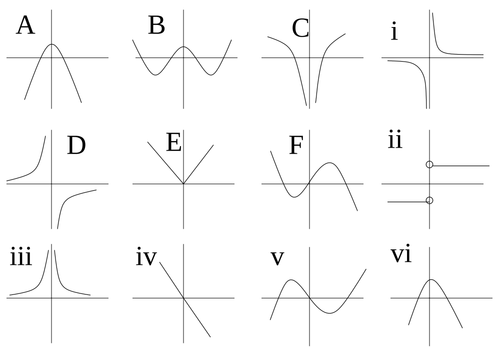
Week in Review # 6

MATH 142 Drost-Fall 2006
Sections 3.4-3.6 Oct 8, 2006

1. Find the derivative of $f(x) = \frac{4}{7x^3} + 5\sqrt[3]{x^2}$ using the power rule.

2. Find the equation of the tangent line to $y = \sqrt{x}$ at the point (9,3).

3. Match the graphs of the functions with the graphs of the derivatives.

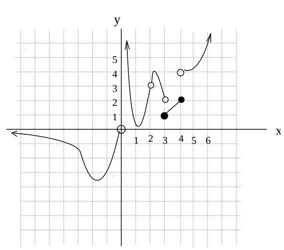


4. Find the following limits, if they exist.

   a) $\lim_{x \to 2} f(x)$

   b) $\lim_{x \to 0^+} f(x)$

   c) $\lim_{x \to -\infty} f(x)$



5. Find the derivative:

   a) $y = 2.4\sqrt{x}$

   b) $y = e^3 + \pi x^2$

   c) $y = 4x^2 - \frac{3}{x} + \pi$

   d) $y = \frac{15x^3 - 40x^2 + 5x - 12}{5\sqrt{x}}$

6. A population of bacteria is growing according to $N(t) = .1t^2 + 5t + 500$ where t is measured in hours. What is the rate of change of the population when t=10?

7. If an object is dropped from a building 185 feet tall, its height above the ground after t seconds is given by

$$s(t) = 185 - 16t^2$$

where $s(t)$ is measured in feet.

   a) Find $v(t)$, the velocity function.

   b) Find $s(1)$ and $s(3)$ and interpret.

   c) Find the average velocity from $t = 1$ to $t = 3$.

   d) When does the object hit the ground?

8. Use the product rule to find the derivative of

$$f(x) = (3\sqrt{x} + 5)(4x^{\frac{3}{2}} - 8)$$

9. If t is the number of weeks after the introduction of a new debugging program for computers, the percentage of the firms in an industry not using the new technology is given by $p(t) = \frac{80 + 5\sqrt{t}}{1 + t^2} + 20$.

   a) What is the initial value?

   b) What is the rate of change of p with respect to t?

   c) What percentage is not using the program after 10 weeks?

10. Given $y = (4x^2 - 5)(7x + 3)(5x - 2)$; find $\frac{dy}{dx}$.

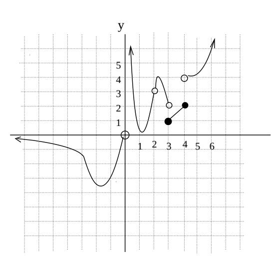
11. Find the derivative of

$$f(x) = \frac{(2x - x^3)\sqrt{x}}{3x + 2}$$

12. Find the derivative of

$$g(x) = (5x - 2)\sqrt[4]{4x^2 + 16}$$

13. Where is the function pictured below discontinuous, and state why it is not continuous at that point.



14. Find the points of discontinuity for the function:

$$f(x) = \begin{cases} -3x + 4, & x < 2 \\ \frac{x^2}{x^2 - 3x - 4}, & x \geq 2 \end{cases}$$

15. For what value(s) of k is the function $f(x)$ continuous over the interval $(-\infty, \infty)$?

$$f(x) = \begin{cases} \sqrt{6-2x}, & x < 1 \\ 3k - 10x, & x \geq 1 \end{cases}$$

Find the derivative of each of the following functions.

16. $f(x) = (3x^2 - 10)^4$

17. $g(x) = (x^2 + 3x)^2(2x+1)^3$

18. $h(x) = \dfrac{x^2 + x + 5}{6x^2 + 30}$

Given $f(5) = -3, f'(5) = 2, g(5) = 1, g'(5) = 4$:

19. Find $p'(5)$ given that $p(x) = f(x) \cdot g(x)$.

20. Find $h'(5)$ given that $h(x) = \dfrac{f(x)}{g(x)}$

ANSWERS:

1. $f'(x) = \dfrac{-12}{7x^4} + \dfrac{10}{3x^{\frac{1}{3}}}$

2. $y - 3 = \dfrac{1}{6}(x - 9)$

3. A) iv, B) v, C) i, D) iii, E) ii, F) vi

4. a) 3, b) ∞, c) 0

5. a) $y' = \dfrac{6}{5\sqrt{x}}$

5. b) $y' = 2\pi x$

5. c) $y' = 8x + \dfrac{3}{x^2}$

5. d) $y' = \dfrac{15}{2}x^{\frac{3}{2}} - 12x^{\frac{1}{2}} + \dfrac{1}{2}x^{\frac{-1}{2}} + \dfrac{6}{5}x^{\frac{-3}{2}}$

6. Bacteria are growing at a rate of 7/hr.

7. a) $v(t) = -32t$

7. b) After 1 second the object is 169 ft above ground, and after 3 seconds the object is 41 ft above ground.

7. c) -32 ft/sec

7. d) $t \approx 3.4$ seconds

8. $f'(x) = 24x + 30x^{\frac{1}{2}} - 12x^{\frac{-1}{2}}$

9. a) 100%

9. b) $p'(t) = \dfrac{5t^{\frac{-1}{2}} - 320t - 15t^{\frac{3}{2}}}{2(1+t^2)^2}$

9. c) 20.9%

10. $\dfrac{dy}{dx} = 560x^3 + 12x^2 - 398x - 5$

11. $f'(x) = \dfrac{\sqrt{x}(12 + 6x - 14x^2 - 15x^3)}{2(3x+2)^2}$

12. $g'(x) = \dfrac{10x^2 - 4x}{(4x^2+16)^{\frac{3}{4}}} + 5\sqrt[4]{4x^2 + 16}$

13. at x=0, f(0) is not defined

13. at x = 2, f(2) is not defined

13. at x=3, $\lim_{x\to 3}$ DNE

13. at x=4, $\lim_{x\to 4}$ DNE

14. discontinuous at $x = 4$ and $x = 2$

15. k=4

16. $f'(x) = 24x(3x^2 - 10)^3$

17. $g'(x) = 2(x^2 + 3x)(2x+1)^2(7x^2 + 17x + 3)$

18. $h'(x) = \dfrac{-6x^2 + 30}{(6x^2 + 30)^2}$

19. -10

20. 14