

Week in Review # 6

MATH 142

Drost-Fall 2008

4.2 Derivatives of Exponential and Logarithmic Functions

1. Find the derivative of $f(x) = e^x - 4 \ln x + 10$
2. Find y' when $y = 4^x - 5 \log_2 x$
3. Find $\frac{dy}{dx}$ when $y = \ln(x \cdot e^x)$
4. Find the equation of the tangent to the curve $f(x) = e^x + 2$ at $x = 0$.
5. A mathematical model for the average grade on exam 2, of a group of students learning their derivative rules is given by $f(x) = 52 \ln x - .001e^x$ on the domain is $(0, 7)$. x represents the number of hours spent each week on calculus, and $f(x)$ represents the students average grade.
 - a. For the student who studies 4 hrs each week, what would this model predict their score would be?
 - b. What is the expected improvement in your grade if you were studying 4 hr/wk and you increase that 1 hr each week?

4.3 Product and Quotient Rule

6. $g(x) = (x^{\frac{2}{3}} + 2x - 5)(x^{\frac{1}{2}} - 7x^{-2} + 6)$, find the derivative.
7. $h(x) = \frac{2x^3 - 1}{x^2 + 3}$, find $h'(x)$.
8. Given $f(w) = \frac{5\sqrt[3]{w}}{w - 2}$, find $\frac{df}{dw}$.
9. Find y' when $y = \frac{\log(x^4)}{\sqrt{x} + \sqrt[3]{x^2}}$. DNS
10. Find the derivative of $f(x) = 8^x \left(\frac{x\sqrt[3]{x}}{\sqrt[5]{x^3}} \right)$

4.4 The Chain Rule

11. $f(x) = (x^2 + 1)^5$ Find $f'(x)$.
12. Find the values of x where the tangent line is horizontal for $f(x) = \frac{x^3}{(4 - x)^5}$.
13. $f(x) = e^{4x^2 - 5x + 2}$ Find $f'(x)$.
14. $f(x) = \sqrt{x^4 + 1} \cdot e^{x^2 + 1}$ Find $f'(x)$.
15. The cost function for Littleton Little League is modeled by $C(x) = \ln(3x^2 + 4x + 10)$. Find $C'(20)$ and interpret. x represents the number of players in the league, and $C(x)$ represents the cost in hundreds of dollars.

3.7 Marginal Analysis

The Wicket Ewok Factory has costs described by the function, $C(x) = 15x + 4\sqrt{x} + 80$ and charges \$5 more than the number ordered. Let x be the number of stuffed animals made and sold.



16. Find the marginal cost.
17. Find the marginal revenue.
18. Find the marginal profit.
19. Find the quantity they need to make and sell to break even.

Hint: Use your calculator to find the intersection of 2 functions.
20. Given $R(x) = 4x^2 - \frac{1}{2}x + 8$, find the marginal average revenue.
21. Given $C(x) = 250 - 5x$, find the average cost for $x = 8$.
22. Solar Energy INC, creates sidewalk lanterns which run on solar energy. They have determined a price demand function for their product of $x = 2500 - 100p$. They have fixed costs of \$504, and variable costs of \$12 per item.
 - a) Find the domain of the price-demand function.
 - b) Find the cost function.
 - c) Find $R(x)$, the revenue function in terms of the quantity produced and find its domain.
 - d) Find the quantity they should produce and sell to maximize profit.

ANSWERS

1. $y' = e^x - \frac{4}{x}$
2. $y' = 4^x(\ln 4) - \frac{5}{x \cdot \ln 2}$
3. $y' = \frac{1}{x} + 1$
4. $y = x + 3$
- 5a. 72%
- 5b. 13%
6. $g'(x) = (x^{\frac{2}{3}} + 2x - 5)(\frac{1}{2}x^{-\frac{1}{2}} + 14x^{-3}) + (x^{\frac{1}{2}} - 7x^{-2} + 6)(\frac{2}{3}x^{-\frac{1}{3}} + 2)$
7. $h'(x) = \frac{2x^4 + 18x^2 + 2x}{(x^2 + 3)^2}$

8. $\frac{-10(w+1)}{3w^{\frac{2}{3}}(w-2)^2}$
9. $y' = \frac{(\sqrt{x} + \sqrt[3]{x^2})\left(\frac{4}{x \ln 10}\right) - (\log x^4)\left(\frac{1}{2}x^{-\frac{1}{2}} + \frac{2}{3}x^{-\frac{1}{3}}\right)}{[\sqrt{x} + \sqrt[3]{x^2}]^2}$
10. $y' = 8^x\left(\frac{11}{15}x^{-\frac{4}{15}} + x^{\frac{11}{15}} \cdot 8^x \cdot \ln 8\right)$
11. $f'(x) = 5(x^2 + 1)^4(2x)$
12. $x = 0, -6$
13. $y' = (8x - 5)e^{4x^2 - 5x + 2}$
14. $y' = 2x \cdot e^{x^2 + 1}[x^4 + 1]^{-\frac{1}{2}}[x^4 + x^2 + 1]$
15. Adding a 21st player will increase the cost by \$9.61.
16. $MC = 15 + \frac{2}{\sqrt{x}}$
17. $MR = 5 + 2x$
18. $MP = 2x - 10 - \frac{2}{\sqrt{x}}$
19. 16 stuffed animals
20. $MAR = 4 - \frac{8}{x^2}$
21. \$26.25
- 22a. $0 \leq p \leq 25$, b. $C = 504 + 12x$, c. $R = 25x - 0.01x^2$, $0 \leq x \leq 2500$, d. $x = 650$