Week in Review # 7

MATH 142 Drost-Fall 2006
Section 3.7-4.2 Oct 15, 2006

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1. Estimate the profits of the next item at $x = 27$ given $P(x) = \sqrt[3]{x}(x - 54)$

2. With $C(x) = 2x^{\frac{5}{2}} + 100$, using the marginal cost, approximate the cost to produce the 4^{th} item.

3. If $p = 50 - 2x$ find the marginal revenue at $x = 3$.

4. Given $h(x) = (f \circ g)(x)$, find possible functions for $f(x)$ and $g(x)$ when

   a. $h(x) = \dfrac{5}{(x+1)^2} + 4x + 7$

   b. $h(x) = 6(3x - 2)^4 + 4(3x - 2)^2 + 9$

(#5-#11) The Campus Comfy Furniture Company determines that the price-demand function for their loft unit is $p(x) = .2x^2 - 14x + 320$ on the interval $0 \leq x \leq 40$. They have fixed cost of \$1000 and variable costs of \$60 per unit.

5. Find the cost function.

6. Find the revenue function.

7. Find the profit function.

8. Find the average profit when 15 units are produced.

9. Find the marginal average cost.

10. Estimate the cost to produce the 12^{th} item.

11. Find MAP(10) and interpret.

Find the derivative of each of the following functions. (#12-#14)

12. $f(x) = (3x^2 - 10)^4$

13. $g(x) = (x^2 + 3x)^2(2x + 1)^3$

14. $h(x) = \dfrac{x^2 + x + 5}{6x^2 + 30}$

15. Where is the function, $f(x) = \dfrac{x+5}{2x}$, increasing or decreasing?

16. The concentration of pain killer in the blood stream t hours after taking the medicine is given by $C(t) = \dfrac{(t-9)^2}{t^2 - 4t + 10}$, where $C(t)$ is measured in mg/ml. How many minutes before the pain killer has reached it's maximum concentration?

17. Find the intervals over which $f(x)$ is increasing when $f(x) = \dfrac{x - 6}{x^2 + x - 6}$

18. Find the equation of the tangent to the curve $f(x) = (4\sqrt{x} - 9)^3$ at $x = 9$.

19. Plasma Plus determines that the price-demand function for their newest 27" screen is $p(x) = \dfrac{-x}{400} + 500$, where x represents the number of screens produced and sold. They have fixed costs of \$1797.75 and it cost the company \$495 to make each screen.

   a. Find the revenue function, $R(x)$, and the cost function, $C(x)$.

   b. Find the profit function, $P(x)$ and find the smallest and largest production levels x so that the company realizes a profit.

   c. Evaluate $P'(500)$ and interpret.

   d. How many should they make and sell to maximize profits?

20. Find the critical values for

   a. $f(x) = -x^3 - 3x^2 + 24x$

   b. $g(x) = \sqrt{3x - 12}$

21. Total costs to care for 12 dogs at the Puppy Palace Day Care is \$246. Food and treats run \$8 per dog. If the price-demand function is $p = -\dfrac{1}{20}x + 25$, find the number of dogs they should care for, to maximize profit.

22. Find the derivative and **DO NOT SIMPLIFY**:

$$y = \frac{(x^3 + 4x^2)\sqrt{3x - 4}}{\sqrt[3]{x^2 + x + 1}}$$

23. Fuel Cells INC has a price-demand function, $x = 75 - .2p$, with fixed costs of \$4600 and each fuel cell costs \$60 to make.

   a. Find the domain of the price-demand function.

   b. Find the marginal cost function.

   c. Find the revenue function and its domain.

   d. Find the break even points.

   e. Find the number of fuel cells they should make and sell to maximize profits.

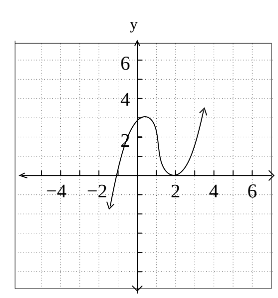
24. From the graph below of $f'(x)$,

   a. Where is $f''(x) > 0$?

   b. Where is $f(x)$ increasing?

   c. Where does $f(x)$ have a relative max or min?

   d. Where does $f'(x)$ have a point of inflection?



Answers:

1. 2

2. \$25.98

3. 38

4a. $f(x) = \dfrac{5}{x^2} + 4x + 3,\ g(x) = x + 1$

4b. $f(x) = 6x^4 + 4x^2 + 9,\ g(x) = 3x - 2$

5. $C(x) = 60x + 1000$

6. $R = 0.2x^3 - 14x^2 + 320x$

7. $P = 0.2x^3 - 14x^2 + 260x - 1000$

8. $AP(15) = 28.\overline{3}$

9. $MAC = \dfrac{-1000}{x^2}$

10. 60

11. At a production level of 10 loft units, the rate of change of *average profit* is zero.

12. $f'(x) = 24x(3x^2 - 10)^3$

13. $g'(x) = 2(x^2 + 3x)(2x + 1)^2(7x^2 + 17x + 3)$

14. $h'(x) = \dfrac{-6x^2 + 30}{(6x^2 + 30)^2}$

15. decreasing over the intervals $(-\infty, 0), (0, \infty)$

16. In 68.5 minutes, the drug has a maximum concentration.

17. $(0, 2), (2, 12)$

18. $y = 18x - 135$

19a. $R(x) = -\dfrac{1}{400}x^2 + 500x,\ C(x) = 1797.75 + 495x$

19b. $P(x) = -\dfrac{1}{400}x^2 + 5x - 1797.75, x = 470, 1530$

19c. The approximate profit on the 501^{st} screen is \$2.50.

19d. They should make 1000 screens to maximize profit.

20a. $x = -4, 2$

20b. $x = 4$

21. 170 dogs

22. $y' = \dfrac{\sqrt[3]{x^2 + x + 1}[(x^3 + 4x^2)(\frac{1}{2})(3x - 4)^{-1/2}(3) + \sqrt{3x - 4}(3x^2 + 8x)] - (x^3 + 4x^2)\sqrt{3x - 4}[(\frac{1}{3})(x^2 + x + 1)^{-2/3}(2x + 1}{(\sqrt[3]{x^2 + x + 1})^2}$

23a. $(0, 375)$ price ranges from \$0 to \$375.

23b. $MC = 60$

23c. $R = 375x - 5x^2$, $(0, 75)$ the quantity ranges from 0 to 75 cells

23d. $x = 23, 40$

23e. They should make 31.5 fuel cells to maximize profit.

24a. $(-\infty, 0), (2, \infty)$

24b. $(-1, 2), (2, \infty)$

24c. at $x = -1$ relative min

24d. at $x = 1$